

## SYNTHESIS AND SELECTION OF OPTIMAL STRUCTURES FOR MACRO-ROBOTS

Penka GENOVA, Kostadin KOSTADINOV, Vladimir KOTEV

**Abstract:** *More and more technological operations become robotized. The present study focuses on problems of structural synthesis and optimization, referring predominantly to macro-robots with closed kinematic chains (CKC), but theory is valid for the structures of micro-robots as well. In addition to the known structural dependencies, the authors have reached two new ones, whereby the synthesis turns from formal permutation of the connection of links with kinematic pairs (KP) into a purposeful creation of KC and the extraction of isomorphic structures. Through the introduction of restrictions and optimum criteria, sets of structures are selected, which are suitable for robot manipulation systems.*

**Key words:** *structural synthesis, selection of optimal structures.*

### 1. INTRODUCTION

Structural analysis is also known in literature as type synthesis or "number synthesis" [1]. A Systematic definition of the optimal type of the mechanisms is necessary. There are many examples of unsuccessful design of mechanisms in industry due to the inappropriate selection of the mechanism type. K. Hein defined the type synthesis as "selection of a mechanism specific type, for instance the selection of linkage or cam mechanisms" [2]. Crossley proposed that applying the method of type synthesis, one should analyze the desired motion in such a manner as to find the simplest mechanisms which would exactly or approximately realize that motion as it is done in [3]. Freudenstein and Dobrjanskyj proposed a more extensive and simpler definition of the type synthesis i.e. "definition of the mechanism structure depending on its kinematic capabilities". This is most appropriate in the cases of micro and nano robots when the functional task is precise defined preliminary for certain application [4]. The robot regional structure, so called macro robot, is synthesized in such manner to fulfill the requirements of the functional task [5].

One may consider the type synthesis as consisting of two main steps that are to be undertaken: determination of the structure of mechanisms and analysis of how effectively they could realize the desired functions after the performance of a dimensional synthesis. The first step is known as number synthesis, while the second one is often called structural analysis [6]. Most generally, the system design of mechanisms comprises three procedures: problem statement, type synthesis and dimensional analysis.

### 2. THEORETICAL FOUNDATIONS

The structure or the type synthesis of plane closed kinematic chains (CKC) is an object of a number of publications. The basic principle of structure generation is permutation of the connection of different n-ary links with a specified class of KP, and they should satisfy the above relations for given degree of freedom (DF). Franke, Crossley, Hein, Manulesku are classical scholars in finding sets of structures. Some researchers use graph images, and the pioneer in this field is Freudenstein (1967). Note here the interesting works of Manulesku in 1970, who deals with structures in which some of the dimensions of the n-ary links can be zero. However, their guidance capabilities increase but at the expense of some structural inconveniences owing to the introduction of a large number of KP with one and the same axis.

Micro-actuators for micro- and nano-robots have been developed during the last decade. The structure of their mechanisms has a number of specific characteristics but the mechanisms subject to the same structural dependencies. Macro-robots are characterized by macro-displacements which are by orders larger than those of the micro-robots. This yields significant differences in the design of KP and selection of actuators.

In compliance with the concepts of type design of mechanisms already specified, we define the main task of the present study, i.e.:

- for the purposes of the structural synthesis to set up a fuller system of Diophantine equations in order to avoid the isomorphic structures;
- to propose criteria for optimal selection of structures for manipulation systems of macro-robots out of known non-isomorphic sets of KC.

We adopt the following structure of our expose:

- we introduce general restrictions on the structures which make them suitable for manipulation systems of macro-robots;
- we submit short description of sets of structures that are to be studied;
- we subject them to criterial estimations;
- we perform hierarchic ordering in accordance with the quantitative and qualitative estimations made;
- we adopt a generalized profit function and classify structures depending on the function values.

The basic relations that all structures obey, including those with open kinematic chains (OKC), are [7]:

$$2p = \sum j n_j, \quad (1)$$

$$n = p - r + 2, \quad (2)$$

$$h = (6 - b)(n - 1) - \sum (k - b)p_k, \quad (3)$$

where  $p$  is the total number of kinematic pairs (KP),  $n_j$  is the number of  $j$ -ary links,  $n$  is the total number of links,  $r$  is the number of closed contours in KC,  $b$  is the number of the restrictions common for KC (for instance,  $b = 3$  for plane KC) and  $k$  is the class of KP depending on the number of limited relative motions ( $k = 1$  to  $5$ ).

There are two structural dependencies, which are very useful. One of them it was published years ago, and we have come up with the second one recently. These dependencies are worked out through the method of induction and are valid for all types of mechanisms. The idea is that all poly-contour mechanisms consist of  $r$  number of mono-contour KC. It turned out that the total number of DF of this KC is an invariant constant, which depends only on DF of the synthesized or analyzed mechanism, and on the number of mono-contours. Analytically recorded the dependency is:

$$\sum h_i = 2h + (6 - b)(r - 2), \quad (4)$$

where  $h_i$  are DF of each mono-contour. It becomes clear from (4) that for each class of KC (plane, spacial, etc.) this parameter depends only on DF of the mechanism and the number of contours. Especially important is the fact that the said parameter is invariant also with regard to the type of KP, i.e the mechanism can be synthesized only with KP of 5<sup>th</sup> class, and then it may be modified on the basis of the principle for kinematic equivalence of KP from a lower class to such of a higher class, and to be realized with another structural and construction design.

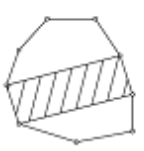
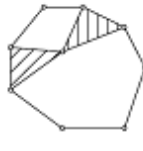
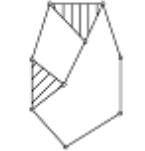

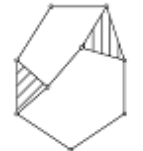
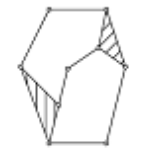
The second dependency is:

$$\sum n_i = 2[h + (6 - b)(r - 1)], \quad (5)$$

where  $n_i$  is the number of links in each mono-contour. This parameter is invariant only with one and the same type of KP, for example of 5<sup>th</sup> class, since every pair of a lower class decreases the number of links. Taking into consideration equation (2) and dependency [8]

Table 1

Structural Synthesis  $h = 3, r = 3, b = 3$

Nr.	1			2			3		
	$h_i$	1-2-6			1-3-5			1-4-4	
$n_i$	4-5-9			4-5-8			4-7-7		
$n_j$	$n_4$	$n_3$	$n_2$	$n_4$	$n_3$	$n_2$	$n_4$	$n_3$	$n_2$
	1	0	7	0	2	6	0	2	6
Scheme of KC									
Nr.	4			5			6		
	$h_i$	2-2-5			2-3-4			3-3-3	
$n_i$	5-5-8			5-6-7			6-6-6		
$n_j$	$n_4$	$n_3$	$n_2$	$n_4$	$n_3$	$n_2$	$n_4$	$n_3$	$n_2$
	0	2	6	0	2	6	0	2	6
Scheme of KC									

$$h = p - (6 - b)(r - 1), \quad (6)$$

then the synthesis is reduced to finding the whole number solutions of equation (6) and (2), and of a system of Diophantine equations (4) and (5) when  $h$ ,  $b$  and  $r$  are specified. Since the solutions of (4) and (5) correspond to the generation of the constituent mono-contours of KC, then the synthesis is reduced to their combining.

This approach allows us to avoid many non-functional, formally generated KC, such as ones with partial (inadequate) DF, with CKC without relevant mobility (these are the 3-link chains with the plane linkage), isomorphic ones and others. In confirmation of the previously mentioned, we will point out that of the generated 97 ten-link structures with 3 DF IMI only four are adequate. The extraction of the remaining 93 is a rather arduous process.

We will illustrate the described algorithm with the synthesis of 8-link KC with three DF at  $b = 3$  and  $r = 3$ . From (6) and (2) we obtain  $p = 9$  and  $n = 8$ . The invariants of (4) and (5) have the values of 9 and 18, respectively. The solutions of (4) are shown in Table 1. There are 6 combinations which correspond to the three mono-contours recorded with the number of their links (for example to  $h_i - 1-2-6$  correspond the contours  $n_i 4-5-9$ ).

Firstly, the contour with the maximum number of links is outlined, and thereafter it is supplemented with the relevant number of missing links and KP. From equation (1) the number of binary, ternary, etc. links is obtained. Comparing the mono-contours to the type of links, it becomes clear that KC 1 and 2 are with partial

DF (they are drawn with the purpose of their visualization). In KC 3 and 4 only two of the actuators can be on fixed link, while in the last two KC, all three actuators can be relatively fixed.

### 3. SELECTION OF OPTIMAL STRUCTURES WITH CKC. GENERAL RESTRICTIONS IMPOSED OVER THE STRUCTURES OF MACRO-ROBOTS

1. The structure under consideration should contain an  $n$ -ary link where  $n$  should be larger than or equal to the DF, and the link should be chosen such as to be a frame. This means that all actuating devices should be carried by or should be directly joined to the relatively fixed link. The effect is dynamic-decrease of movable masses results in the decrease of inertial forces. For instance, there should be at least one ternary link in the structure of a manipulation system (MS) with three DF. This restriction may also be introduced as a criterion with a suitable quantitative estimation;

The structure should contain a link with motion corresponding to the DF of the mechanisms, which should be chosen to be an executive one. For instance: in a 7-link KC with two DF there are two options Fig.1 if the ternary link is selected for a frame, namely: when driven links (DL) are 2 and 3 then output links (OL) can be only links 6. When DL- 2 and 4 then OL can be 4 or 5 or 6.

2. There should be no parasitic contours with local DF in the structure, since the KC disintegrate into more elementary ones.

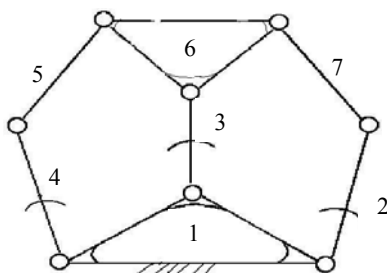


Fig.1. Seven links mechanism.

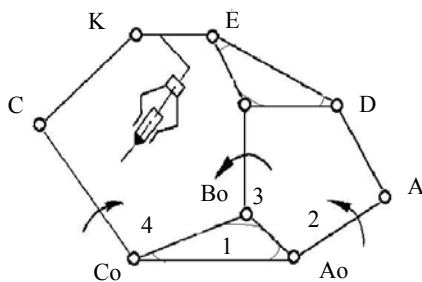


Fig.2. Eight links mechanism.

3. Choose from structures with three (plane) and more DF such structures, which would allow decoupling of the inverse problems of kinematics with respect to position and orientation. Fig.2 shows a 8-link mechanism with three DF consisting of two internal contours -  $A_0ADBB_0$  (quintuple) and  $B_0BEKCC_0$  (sextuple). If links 2 and 3 are driver links, then the position of point E is defined. Via the third actuation 4 we define the position of point C. Via point E and point C we find the position of point K, i.e. the position of orientation EK or EB. In this sense the two inverse problems are decoupled.

### 4. CRITERIA FOR THE ESTIMATION OF CKC FOR MACRO-ROBOTS

We adopt the following criteria for the estimation of KC for use in manipulation systems (MS) of macro-robots:

$$K_1 = j/h, \quad (7)$$

where the  $j$ -ary links are denoted by  $j$ , and the DF-by  $h$ . When a link with  $j \geq h$  is present, then  $K_1 = 1$ . This criterion is valid for structures which comply with restriction 1;

$$K_2 = h/n, \quad (8)$$

$$K_3 = n/b, \quad (9)$$

where  $n$  is the number of links and  $b$ - the number of ribs. For links with more than two KC, we have,  $b > n$ . For mono-contour KC however, we have  $b = n$ , i.e. the maximal value of  $K_3$  is  $K_3 = 1$ , which we consider to be the optimal value. The presence of ribs whose number is larger than that of the links increases the weight and overall dimensions, decreases the guidance capability, but improves carrying capacity and stability of the structure. Hence, this criterion should be used complying with the robot functions;

$$K_4 = 1 - [\sum s p_s] / p, \quad (10)$$

where  $p_s$  is the number of axes which coincide with each other  $s$ -times (double, triple etc. hinges). We find that such accumulation of hinges complicates the structure. Obviously, the optimal value is 1;

$$K_5 = 1 - d_s / h, \quad (11)$$

where  $d_s$  is the number of co-axial actuating devices (AD). The optimal value is 1;

$$K_6 = 1 - d_p / h, \quad (12)$$

where  $d_p$  is the number of AD on moving links. For that criterion, the optimal value is also 1.

The system is completely open - one can add or disregard criteria. Note that, (i) - the criteria introduced deal with quantitative estimations and (ii) - the optimal values provided by them are identical, which enables one to design various purposeful complex criteria with equal-in-rights and weighted values. This fact also facilitates the subjective selection of the weighting factors (effect factors).

Furthermore, yet on structural level criteria for estimation of the degree of complexity of reverse kinematic task may also be introduced. Without pretending for absolute precision we propose that the estimation is performed according to the following two criteria depending on the number of the dependent equations and the mechanism class:

$$K_7 = 1 - u / u_{\max}, \quad (13)$$

$$K_8 = 1 - c / c_{\max}, \quad (14)$$

wherein  $u$  and  $c$ ,  $u_{\max}$  and  $c_{\max}$ , respectively denote the number of the equations and the mechanism class, as well as their maximum values for the selected set. The first 6 criteria are typically structural, while the last two ones are rather kinematic.

## 5. SELECTION OF OPTIMAL STRUCTURES.

### HIERARCHIC MONO- AND POLYCRITERIAL ORDER OF STRUCTURES

We select a set KC with equal number of DF. In compliance with the above said restrictions we specify which links will be frame, input and output links. One and the same KC, depending on the selection of these links, have different qualities. Each representative of a set of structures is assessed according to each one of the criteria. We introduce multi-criterial profit function. It is most often additive of the type

$$K_{\Sigma} = \sum \lambda_j K_j, \quad (15)$$

where  $\lambda_j$  are subjectively selected weighting factors, assigned by the users according to the purpose of the MS.

A table, where the mechanisms are hierarchically ordered, is prepared illustrating which are the favorites according to a certain criterion, and which are according to the complex estimations. To some extent, the mono-criterial estimations give a good orientation for the selection of weighting factors. Of great importance are the dependencies we proposed for the quantitative and weighted values of the criteria where the comparison of the results is objective. It is possible to introduce qualitative criteria as well, to which the optimal and intermedi-

Table 2

### Values of the kinematic and complex criteria

K/Nr.	1	2	3	4	5
K7	0.4	0.2	0.25	0.4	0.45
K8	0.67	0.33	0.33	0.33	0.33
K7+k8	1.07	0.53	0.58	0.73	0.78
$\Sigma K$	5.74	4.74	4.91	5.28	5.45
K/Nr.	6	7	8	9	10
K7	0	0	0	0	0.2
K8	0.33 / 0	0.33	0.33	0.33	0
K7+k8	0.33 / 0	0.33	0.33	0.33	0.2
$\Sigma K$	5.1 / 4.77	5.26	5.26	5.26	5.66
K/Nr.	11	12	13		
K7	0.25	0.2	0.25		
K8	0.33	0.33	0.33		
K7+k8	0.58	0.53	0.58		
$\Sigma K$	5.58	5.36	5.45		

ate values are subjectively assigned. Such criteria are for e.g. the complexity of structure or production technology, market value or search, etc.

The mentioned algorithm is illustrated with a particular example (Table 3). We selected an example set of KC with three DF – open Nr. 1, hybrid (4, 5 and 6-link with numbers from 2 to 9) and closed ones - Nr. 10 to 13. We assume, where it is possible, that the dimensions of the ternary links in KC are zero. In the first three columns are given the numbers of links «frame, inputs and outputs». In the next columns are presented the calculated values of the 6 structural criteria and their corresponding complex criterion. Table 2 shows the values of the two kinematic criteria, their sum total and the sum of the structural and kinematic estimations. The results analysis shows:

1. The hybrid structures with 4-link closed KC (Nr. 2 and 3) have the lowest complex estimation, and the highest one is of link KC (Nr.10 and 11);
2. comparatively, the first and the sixth criterion are the most influential;
3. The values of the kinematic criteria ( $K_7$  and  $K_8$ ) vary within rather broad limits, which means that their effect is considerable. First comes open structure

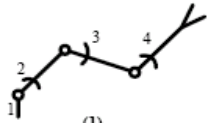
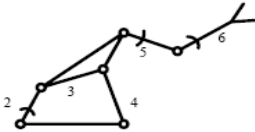
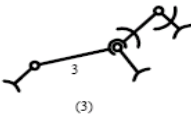
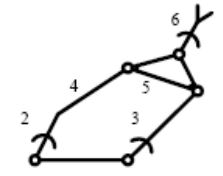
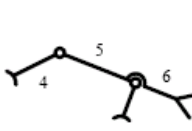
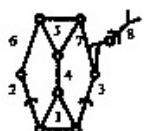
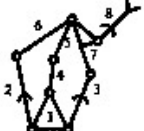

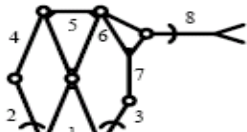
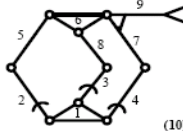
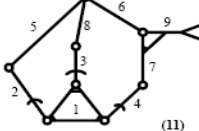


Nr.1, but again structures 10 and 11 follow it very closely. Structures 5 and 13 show good results too, and 12, 4 and 7 are close to them. Obviously, on structural level the competition is cruel.

## 6. CONCLUSIONS

One has to consider the geometrical and the static, dynamical and economic criteria to select a structural favourite when the criteria are equal-in-rights. The selection of structure for the different levels is considerably

Table 3

Hierarchical order according  $k_1 + k_6$  and  $\Sigma K_i$

Scheme	Links		Criteria							
	Fr	In	Out	$K_1$	$K_2$	$K_3$	$K_4$	$K_5$	$K_6$	$\Sigma K$
 <p>(1)</p>	1	2 3 4	4	1/3	1	1	1	1	1/3	4,67
 <p>(2)</p>  <p>(3)</p>	1	2 5 6	6	2/3	3/6	5/7 5/6	1	1	1/3	4,21 4,33
 <p>(4)</p>  <p>(5)</p>	1	2 3 6	6	2/3	3/6	5/7 5/6	1 5/6	1	2/3	4,55 4,67
 <p>(6)</p>  <p>(7)</p>  <p>(8)</p>	1 1 1	2 3 2 3 2 3	7 7 7	1 1 1	3/8	8/1 1 8/9 8/9	1 1	1 1	2/3	4,77 4,93 4,93
 <p>(9)</p>	1	2 3	6	1	3/8	8/1 1	1	1	2/3	4,76
 <p>(10)</p>  <p>(11)</p>	1 1	2 3 4 2 3 4	7 7	1	3/9	9/1 2 9/1 1	1 9/1 0	1 1	1	5,08 5,05
 <p>(12)</p>  <p>(13)</p>	1 1	2 3 4 2 3 4	7 7	1	3/9	9/1 2 9/1 1	9/1 2 9/1 1	1 9/1 0	1	4,83 4,87

facilitated when there are specified priorities (weighting factors).

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