Proceedings in Manufacturing Systems, Vol. 5 (2010), No. 4 ISSN 2067-9238

FINITE ELEMENT ANALYSIS OF VIBRATING BEAM STRUCTURAL ELEMENTS WITH TRANSVERSAL CRACK

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Abstract: *The existence of a crack in structural elements introduces local modification of flexibility and consequently affects the dynamic behaviour of the cracked element. In this paper, the modal properties such as the natural frequencies and eigenvectors for bending vibrations of cracked beams are studied. The focus of this paper is on the modal analysis of a beam structural element with a transverse single-edge open crack. These modal properties are obtained using the finite element method. The influence of the geometrical parameters of the crack, (especially the position and depth of the transversal crack) on the modal properties is examined. Changes of the modal properties due to the crack appearance are presented.*

Key words: *beam structure, bending vibration, transversal crack, natural frequency, FEM.*

1. INTRODUCTION

The requirements on design and production of the constructions, equipment and tools with higher performance, materially and economically effective became higher in the recent years. The increase of performance is mostly achieved by increasing of operating velocities, speeds and cycles. All these requirements and changes affect the dynamical behaviour of a mechanical system. These mechanical systems are often loaded by external dynamical effects, which produce undesirable structural and dynamic difficulties and overloading of structural elements.

The structures under operation loading are subjected to degenerative effects that may cause initiation of structural defects such as cracks which, in time lead to the failure or breakdown of the structure. The cracks or other structural defects affect the dynamic behaviour of structures and the change of stiffness and damping properties. When a structural element is cracked, the cracks induce a local modification of structural flexibility which is a function of the crack depth, thereby changing its dynamic behaviour and its stability characteristics. The presence of a crack in the structure could not only cause a local variation of the stiffness but it also affects the mechanical behaviour of the entire structure to a considerable extent.

The presence of a crack in vibrating structures, could lead, in time, to catastrophic failure of the structure. In order to improve the safety, reliability and operational life, it is important to ensure the integrity of structural elements. The cracks or other defects in a structural element influence its dynamical behaviour and change its stiffness and damping properties. Cracks in a vibrating structure modify its natural frequencies because it becomes more flexible [1]. The natural frequencies and mode shapes of the structure contain information about the location and dimensions of the damage.

The vibration characteristics of structures can be useful for an on-line detection of defects such as cracks [2], offering thus an effective, inexpensive and fast means of "non-destructive testing“ without actually dismantling the structure. The analysis to detect, locate and quantify the extent of damages in beams from the knowledge of their vibratory characteristics falls within what is known as “inverse problems”. To address the solutions of an inverse problem it is necessary to known the solutions of the named “direct problem”, i.e. as a first, the natural frequencies and mode shapes of cracked beams for the defined parameters of crack must be determined. The effects of location and dimensions of a crack on the modal properties of cracked beam structures are obtained and analysed in this paper.

It is well-known that the beam is one of the most important structural elements, which is very often used in equipments and constructions. The effect of crack on the dynamic properties of beams has received much attention because of its importance in mechanical, automotive, civil, shipbuilding and aerospace engineering applications. In particular, the natural frequencies and mode shapes [3 and 4] of cracked beams can provide an insight into the extent of damage.

As representative models of the cracked structural beam elements with a transversal edge crack and with the following boundary conditions:

- clamped-free (C-F),
- clamped-clamped (C-C),

are studied in this paper.

The modal analysis approach is used to formulate the corresponding eigenvalue problems. The effect of two of the crack parameters on the dynamic behaviour of the cracked beam under consideration is investigated. These two parameters are the depth and position of the crack.

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2. FORMULATION OF THE PROBLEM

The problem considered here is a beam structure with transversal crack as it is shown schematically in Fig. 1. The beam structure of the length l_b has a uniform rectangular cross-section (width − b_b, height − h_b). It is supposed that the beam structure has the crack of depth h_c located at a distance l_c from the right-side of the beam. It is assumed that the crack has a uniform depth across the width of the beam structure. Only fully open cracks are considered.

The mathematical model used for modal analysis of considered beam structure [6] is based on the finite element method. A three-dimensional finite element model of the beam with transverse non-propagating crack is used to the analysis of the crack effect on the modal properties of given beam structures.

Generally, the FE equation of motion for a free vibration of beam structure without crack has the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0\,. \tag{1}$$

where **M** - mass matrix, **K** - stiffness matrix, **q** - vector of nodal displacements, $\ddot{\mathbf{q}}$ - vector of nodal accelerations.

The basic modal properties (natural angular frequencies, eigenvectors) of a beam without crack, can be obtained from the solution of following eigenvalue problem

$$(\mathbf{K} - \omega^2\mathbf{M})\boldsymbol{\phi} = \mathbf{0}\,. \tag{2}$$

where ω - natural angular frequency, $\boldsymbol{\phi}$ - eigenvector.

Both the initiation and growth of the crack in beam structural element affect the natural frequencies and eigenvectors of cracked beam. It must be noted that the mass distribution of cracked beam is not changed, but stiffness of this beam is considerably changed. When the crack is considered, the stiffness matrix of the beam has to be changed.

The equation of motion [1], [5] for a free vibration of cracked beam has the form

$$\mathbf{M}\ddot{\mathbf{q}}_c + \mathbf{K}_c\mathbf{q}_c = \mathbf{0}\,. \tag{3}$$

where $\mathbf{K}_c$ − stiffness matrix of the cracked beam, $\mathbf{q}_c$ − nodal displacement, $\ddot{\mathbf{q}}_c$ − nodal accelerations.

The eigenvalue problem for equation (3) can be written

$$(\mathbf{K}_c - \omega_c^2\mathbf{M})\boldsymbol{\phi}_c = \mathbf{0}\,. \tag{4}$$

where ω_c − natural angular frequency of the cracked beam, $\boldsymbol{\phi}_c$ − eigenvector of the cracked beam.

The loss of integrity of the beam cross-section causes a change in bending stiffness of the beam, i.e. the impact of transverse crack is represented by a loss of connection between nodes of the model. Then, the changes in matrix stiffness are represented by changes in the value of the element stiffness matrix corresponding to the position of the cracks in the beam body. When in the mathematical model (see Eq's (3) (4)) the change of stiffness matrix of beam with transverse crack occurs, and its weight does not change, also the change of modal properties (natural frequencies ω_c and mode shapes $\boldsymbol{\phi}_c$) can be expected.

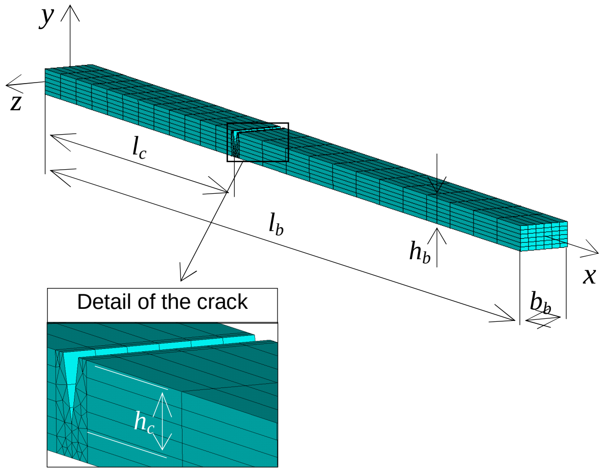

Fig. 1. Finite element model of beam structural element with transversal crack.

The modal analysis of beam structural elements with transversal crack are carried out by using FEM code ANSYS. The 3D SOLID95 finite element from the standard elements of the ANSYS code database was used to create computational beam models.

3. NUMERICAL RESULTS

The effect of the position and size parameters of transversal crack on the fundamental modal properties of cracked beam structures is investigated. The natural frequencies of clamped-free (C-F) and clamped-clamped (C-C) beams are studied.

The analyzed beams have the following geometrical dimensions: uniform rectangular cross-section with width b_b =12 mm and height h_b = 6 mm, length l_b = 450 mm for C-F beam, l_b = 1000 mm for C-C beam. The material properties of the both considered beam structures are: Young's modulus E = 210 GPa, Poisson´s ratio μ = 0.3 and mass density of the beam material ρ = 7800 kgm^{-3}. The value of the crack depth is considered in the range h_c = 1÷5 mm.

The geometrical parameters of the crack are specified by using of the following non-dimensional crack parameters

- non-dimensional crack position

$$\xi_c = \frac{l_c}{l_b} \in \langle 0.0; 1.0\rangle\,, \tag{5}$$

- non-dimensional crack depth

$$\delta_c = \frac{h_c}{h_b} \in \langle 0.0; 0.83\rangle\,. \tag{6}$$

The non-dimensional natural frequency for the k^{th} mode shape is introduced and it is defined as a frequency ratio

$$\vartheta_k = \frac{f_{ck}}{f_k}\,, \tag{7}$$

where

f_k - k^{th} natural frequency of the beam without crack,
f_{ck} - k^{th} natural frequency of the cracked beam.

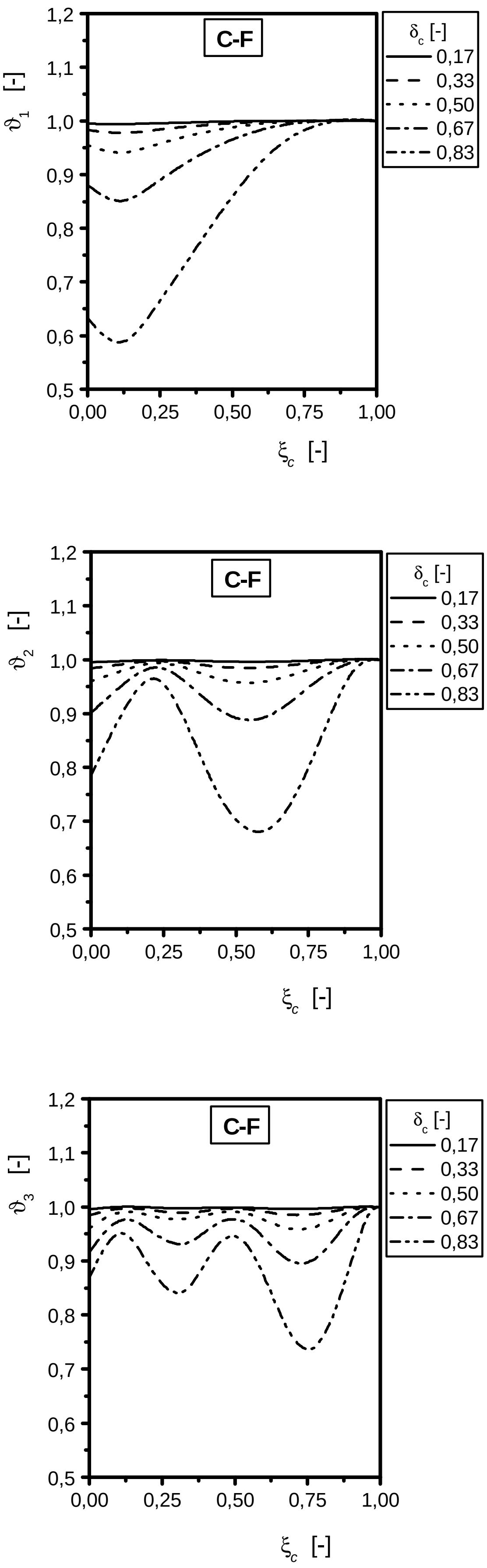

Fig. 2. The first three natural frequencies ϑ_k (C-F beam) on crack position ξ_c, for different δ_c.

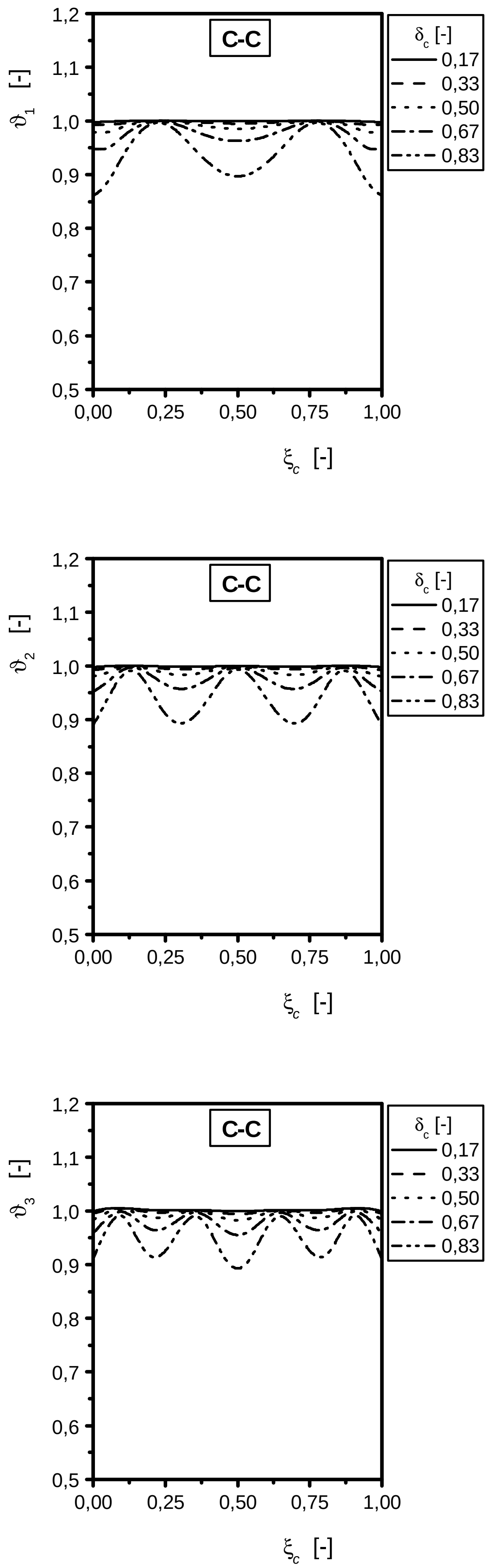

Fig. 3. The first three natural frequencies ϑ_k (C-C beam) on crack position ξ_c, for different δ_c.

The dependencies of the first three non-dimensional natural frequencies ϑ_k of C-F beam on non-dimensional crack position ξ_c for the different non-dimensional crack depths δ_c are shown in Fig. 2. The curves for the first natural frequency have minimum for value $\xi_c \approx 0.1$ and maximum of the first natural frequency is for ξ_c tend to the free end of beam. Similar situations occur for higher natural frequencies of bending vibration of C-F cracked beam. These curves have more local minima and maxima, respectively. Number of local extremes (maximum, minimum) of the curve depends on the order of the particular natural frequency.

Next, the dependencies of the first three non-dimensional natural frequencies ϑ_k of C-C beam on the non-dimensional crack position ξ_c and for the different crack depths δ_c are shown on Fig. 3.

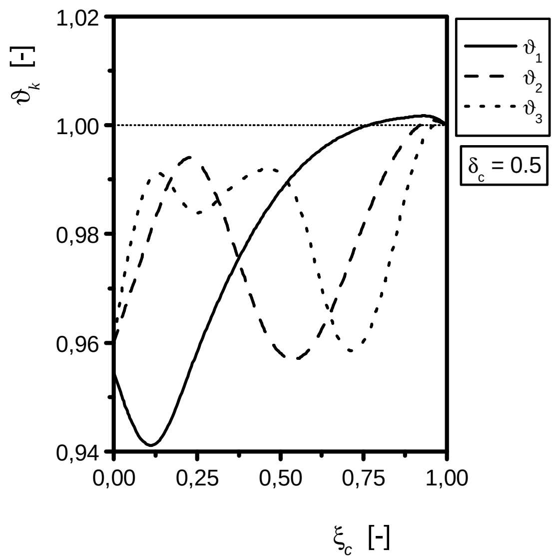

Fig. 4. The first three natural frequencies ϑ_k *vs*. crack position ξ_c ($\delta_c = 0.5$, C-F).

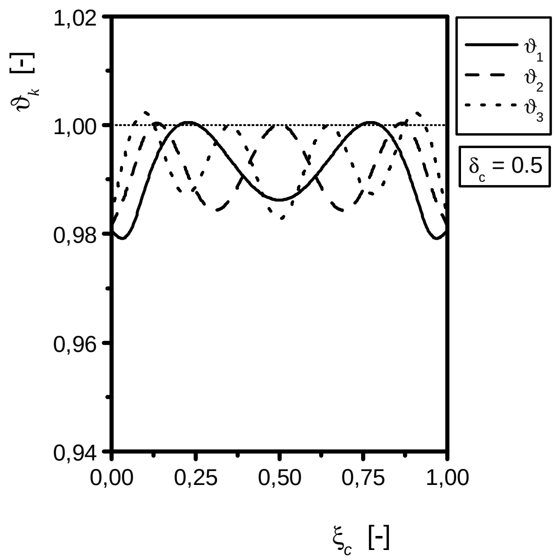

Fig. 5. The first three natural frequencies ϑ_k *vs*. crack position ξ_c ($\delta_c = 0.5$, C-C).

The comparison of the influence of the crack position ξ_c on the first three non-dimensional natural frequencies ϑ_k of the cracked beam can be deduced from the Fig.4 (C-F beam) and in Fig. 5 (C-C beam). The results in Figs. 4 and .5 are for non-dimensional crack depth $\delta_c = 0.5$.

4. CONCLUSIONS

In the present paper, the influence of position and geometrical dimensions of transversal crack on natural frequencies of C-F beam and C-C beam are analysed. The crack position and growth of crack depth cause notable modification of natural frequencies of bending vibration of beam structures. The value of natural frequency depends on the crack position. From the obtained results it follows that each of the curves describing the dependency of k^{th} natural frequency on the crack position has the exact number of local extremes:

- $(k + 1)_{min}$ and $(k)_{max}$ – C-F beam,
- $(k + 2)_{min}$ and $(k + 1)_{max}$ – C-C beam.

The maximal decrease of natural frequencies occurs for the first natural frequencies. For the C-F beam, the first natural frequency is decreasing with about 40% (crack depth $\delta_c = 0.83$, crack position $\xi_c \approx 0.1$). The decrease of the first natural frequency of C-C beam is up to 24% (crack depth $\delta_c = 0.83$, crack position $\xi_c \approx 0.0$ or $\xi_c \approx 1.0$).

Finally, we can say that the crack is a damage that often occurs in structural elements and may cause serious failure of the mechanical systems. Consequently, the crack must be detected in the early state. It can be said that the obtained results are generally valid for considered beam structural elements. The knowledge is applicable for detection and localization of cracks in structures [2].

ACKNOWLEDGEMENTS: The work has been supported by the grant projects VEGA-1/0256/09.

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