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PREFERENCES, UTILITY FUNCTION AND CONTROL DESIGN OF COMPLEX PROCESSES

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Abstract: *Expected utility theory is one of the approaches for assessment and utilization of qualitative conceptual information. The expected utility approach allows for the expert preferences to be taken in consideration in complex biotechnological systems and problems. The expert values are not directly oriented towards the particular problem and as a result, people express substantial uncertainty about their preferences. The topic of this article is recurrent stochastic algorithms for evaluation of expert utilities. A prototype of a value-driven decision support system is discussed. The dialogue between the expert and the computer is modeled numerically. An example of complex control design based on the evaluated utility is demonstrated*.

Key words: *preferences, expected utility, stochastic approximation, complex process, optimal control*

1. INTRODUCTION

The elaboration and the utilization of models of human behavior and the incorporation of human preferences in complex systems are a contemporary trend in the scientific investigations. The aim is to develop decision making with a merger of empirical knowledge (subjective preferences) with the mathematical exactness. People preferences contain characteristic of uncertainty due to the cardinal type of the empirical expert information. The appearance of this uncertainty has subjective and probability nature. Decision making under uncertainty is addressed in mathematics by Probability theory and expected Utility theory. These two together are known as decision theory. The Utility theory deals with the expressed subjective preferences.

The necessity of a merger of empirical knowledge with mathematical exactness causes difficulties. Possible approach for solution of these problems is the stochastic approximation [10 and 11]. The uncertainty of the subjective preferences could be taken as a noise that could be eliminated as typical for the stochastic approximation procedures.

The objective of this paper is to present comfortable tools and mathematical methodology that are useful for dealing with the uncertainty of human behavior and judgment in complex control problems. An example is presented as a mathematical description of the system “technologist, fed-batch cultivation process”. The dialogue “decision maker (DM) – computer” realizes a machine learning on the base of the DM’s preferences.

2. PREFRENCES BASED UTILITY, FORMULATIONS AND EVALUATION

Standard description of the utility function application is presented by Fig. 1. There are a variety of final results that are consequence of the expert or DM activity

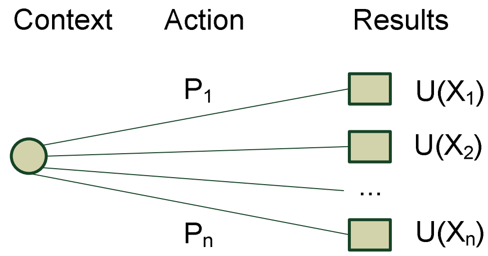

Fig.1. Utility function application.

and choice. This activity is motivated by a technological objective which possibly includes economical, social, ecological or other important characteristics.

A utility function u(.) assesses each of this final results (x_i, $i = 1 \div n$). The DM’s judgment of the process behavior based of the DM’s choice is measured quantitatively by the following formula:

$$u(p) = \sum_i p_i u(x_i),\ \ p = (p_1, p_2, .., p_i, .. p_n), \sum_i p_i = 1. \quad (1)$$

We denote with p_i subjective or objective probabilities which reflect the uncertainty of the final result.

The strong mathematical formulation is the following. Let **Z** is a set of alternatives and **P** is a subset of discrete probability distributions over **Z**. A utility function is any function u(.) which fulfils:

$$(p \succ q,\ (p, q) \in \mathbf{P}^2) \Leftrightarrow (\int u(.)dp > \int u(.)dq),\ (p, q) \in \mathbf{P}^2. \quad (2)$$

The DM’s preference relation over **P** (**Z**$\subseteq$**P**) is expressed by ($\succ$). Its induced indifference relation (~) is defined thus: $((x \sim y) \Leftrightarrow \neg((x \succ y) \vee (x \prec y)),\ (x, y) \in \mathbf{Z}^2)$. We denote with ($\int u(.)dp$) integration based on the probability measure p.

The existence of an utility function $u(.)$ over **Z** determines the preference relation ($\succ$) as a negatively transi-

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tive and asymmetric one [2 and 10]. We mark the lottery "appearance of the alternative x with probability α and appearance of the alternative y with probability $(1-\alpha)$" as $<x, y, \alpha>$. It is assumed that an utility function u(.) exists and that is fulfilled $((q, p)\in \mathbf{P}^2 \Rightarrow (\alpha q+(1-\alpha)p)\in \mathbf{P}$, for $\forall\alpha \in[0,1])$. These conditions determine the utility function with precision up to an affine scale (interval scale), $\mathbf{u_1(.)\sim u_2(.)\Leftrightarrow u_1(.)=au_2(.)+b,\ a>0}$ [2, 6, 10].

The following notations will be used

$\mathbf{A_u=\{(\alpha,x,y,z)/(\alpha u(x)+(1-\alpha)u(y))>u(z)\}}$ and

$\mathbf{B_u=\{(\alpha,x,y,z)/(\alpha u(x)+(1-\alpha)u(y))<u(z)\}}$.

The next proposition is useful.

PROPOSITION 1: If $A_{u1}=A_{u2}$ than $u_1(.)=au_2(.)+b$, $a>0$ [10, 11]**.**

The expected DM utility is constructed by pattern-recognition of $\mathbf{A_u}$ and $\mathbf{B_u}$ [10]. ***The following presents the procedure for evaluation of the utility functions:***

The DM compares the "lottery" $<x,y,\alpha>$ with the alternative z, $z\in Z$ ***("better-$\succ$, $f(x,y,z,\alpha)=1$", "worse-$\prec$, $f(x,y,z,\alpha)=-1$" or "can't answer or equivalent-$\sim$, $f(x,y,z,\alpha)=0$",*** *f(.) denotes the qualitative DM answer* ***).*** *The DM relates the "learning point" (x,y,z,α)) to the set $\mathbf{A_u}$ with probability $D_1(x,y,z,\alpha)$ or to the set $\mathbf{B_u}$ with probability $D_2(x,y,z,\alpha)$. The probabilities $D_1(x,y,z,\alpha)$ and $D_2(x,y,z,\alpha)$ are mathematical expectation of f(.) over $\mathbf{A_u}$ and $\mathbf{B_u}$, respectively, $D_1(x,y,z,\alpha)=M(f/x,y,z,\alpha)$, if $M(f/x,y,z,\alpha)>0$, $D_2(x,y,z,\alpha)=-M(f/x,y,z,\alpha)$, if $M(f/x,y,z,\alpha)<0$. Let $D'(x,y,z,\alpha)$ is the random value: $D'(x,y,z,\alpha)=D_1(x,y,z,\alpha)$, $M(f/x,y,z,\alpha)>0$; $D'(x,y,z,\alpha)=-D_2(x,y,z,\alpha)$, $M(f/x,y,z,\alpha)<0$; $D'(x,y,z,\alpha)=0$, $M(f/x,y,z,\alpha)=0$. We approximate $D'(x,y,z,\alpha)$ by a function of the type $G(x,y,z,\alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$, where $g(x)=\sum_i c_i\Phi_i(x)$*

and $(\mathbf{\Phi}_i(x))$ is a family of polynomials. Then the function g(x) is an approximation of the utility u(.).

The function f(.) (DM answers) fulfills the following conditions [10]:

$f=D'+\xi$, $M(\xi/x, y, z, \alpha) = 0$, $M(\xi^2/x, y, z,\alpha) < d$, $d\in\mathbf{R}$. (3)

It is assumed that u(.) is a "summable" function and that $u(x)\underset{L_2}{=}\sum_i r_i\Phi_i(x)$, $r_i\in\mathbf{R}$, where $(\mathbf{\Phi}_i(x))$ is a family of polynomials. The following notations (based on $\mathbf{A_u}$**)** will be used: $\mathbf{t} = (x, y, z,\alpha)$, $\psi_i(t) = \psi_i(x, y, z, \alpha) = \alpha\Phi_i(x)+(1-\alpha)\Phi_i(y)-\Phi_i(z)$.The next stochastic algorithm realizes the evaluation procedure [10, 11]:

$$c_i^{n+1}=c_i^{\,n}+\gamma_n\left[f(t^{n+1})-\overline{(c^n,\Psi(t^{n+1}))}\right]\Psi_i(t^{n+1}) \quad (4)$$
$$\sum_n \gamma_n = +\infty,\ \sum_n \gamma_n^2 < +\infty,\ \forall n, \gamma_n > 0\,.$$

The coefficients c_i^n take part in the decomposition $g^n(x)=\sum_{i=1}^{N}c_i^n\Phi_i(x)$ and $(c_i^n,\psi_i(t))$ is the scalar product $(c^n,\Psi(t))=\alpha g^n(x)+\underline{(1-\alpha)g^n(y)}-g^n(z)=G^n(x,y,z,\alpha)$.

The line above $\overline{y}=(c^n,\Psi(t))$ means $\overline{y}=1$ if $y > 1$, $\overline{y}=-1$ if $y < -1$ and $\overline{\mathbf{y}}\mathbf{=y}$ if $-1 < y < 1$. The function $G^n(x, y, z, \alpha)$ is positive over $\mathbf{A_u}$ and negative over $\mathbf{B_u}$ depending on the degree of approximation of $D'(x,y,z,\alpha)$. The function $\boldsymbol{g^n(x)}$ ***is the approximation of the empirical DM utility.*** The convergence of the procedure is analyzed in [10]:

THEOREM: Let $(t^1, ..., t^n,...)$ is a sequence of independent random vectors $t^n = (x, y, z, \alpha)$ (procedure (4)) set with distribution F_m and let the sequence of random values $(\xi^1,\xi^2,..,\xi^n,..)$ satisfies the conditions: $M(\xi^n/(x,y,z,\alpha),c^{n-1}) = 0$, $M((\xi^n)^2/(x,y,z,\alpha),c^{n-1}) < d$, $d\in R$ (formula (3)). Let the norm of $\Psi(t)$ is limited by a constant independent from t. The next convergence follows from the recurrent procedure (4):

$$J_{D'}(G^n(x,y,z,\alpha))=M\Big(\int_{D'(t)}^{G^n(t)}(\overline{y}-D'(t))dy\Big)=$$
$$\int\Big(\int_{D'(t)}^{G^n(t)}(\overline{y}-D'(t))dy\Big)dF_m\xrightarrow[n]{p.p}\inf_{s(t)}\int\Big(\int_{D'(t)}^{S(t)}(\overline{y}-D'(t))dy\Big)dF_m.$$

Here p.p. denotes "almost sure" and M is mathematical expectation. The functions $S(t)=\alpha s(x)+(1-\alpha)s(y)-s(z)$ belong to L_2 (defined by the probability measure of F_m). The convergence in other notations is

$$\left\|D'(x,y,z,\alpha)-\overline{G^n}(x,y,z,\alpha)\right\|_{L_2}\xrightarrow{p.p}\min.$$

The proof bases on the "extremal" approach of the potential functions method (kernel trick) [10, 11].

This stochastic evaluation limits the so called *certainty effect* and *probability distortions* identified by *Kahneman* and *Tversky* [5]. In addition, utility dependence on probability can be assessed directly with the proposed procedure. For this purpose we can search for an approximation of the kind $u(x,\alpha)$, $\alpha\in$ **[0, 1], x$\in$ Z,** following *Kahneman* and *Tversky*. The explicit formula of the utility function in this case is:

$$u(x)=\int_0^1 u(x,\alpha)d\alpha\,.$$

The learning points $((x,y,z,\alpha), f(x,y,z,\alpha))$ are set with a pseudo random Lp_τ sequence [10, 13].

The proposed assessment procedure and its modifications are a machine learning approach [10]. The computer is taught to have the same preferences as the DM. Our experience is that the DM is comparatively quick in learning to operate with the procedure (128 learning points and DM answers for about 45 minutes).

3. PREFERENCES AND UTILITY EVALUATION OF A COMPLEX CULTIVATION PROCESS

The complexity of the biotechnological fermentation processes makes difficult the determination of the optimal process parameters. The incomplete information usually is compensated with the participation of imprecise human estimations. Our experience is that the human estimation of the process parameters of a fermentation process contains uncertainty in the frames of 10% to 25%. Here is proposed a mathematical approach for elimination of the uncertainty in the DM preferences and for precise evaluation of the optimal specific growth rate of a fed-batch fermentation process.

The specific growth rate of the fed-batch processes determines the nominal technological conditions [8]. The fed-batch fermentation process is dynamically described by the model of Monod-Wang [8, 14]:

$$\begin{aligned}
\dot{X} &= \mu X - \frac{F}{V} X, \\
\dot{S} &= -k\mu X + (So - S)\frac{F}{V}, \\
\dot{\mu} &= m\left(\mu_m \frac{S}{(K_S + S)} - \mu\right), \qquad (5) \\
\dot{V} &= F, \\
\dot{E} &= k_2\, \mu E - \frac{F}{V} E.
\end{aligned}$$

Where, the biomass concentration is noted with (X) - [g/l]. The substrate concentration is noted with (S) - [g/l] and (S_o) denotes the substrate concentration in the feed - [g/l]. With ($V(t)$) is noted the volume at moment (t) - [l], V_{max} denotes the bioreactor volume and (F) is the substrate feed rate (control input) - [h^{-1}]. The specific growth rate is noted with μ and μ_m denotes the maximum specific growth rate - [h^{-1}]. The constant K_S is the saturation constant - [g/l] and k, k_2 are yield coefficients - [g/g]. The system operation conditions were fixed by the following set of values: μ_m=0.59 [h^{-1}], K_s=0.045 [g/l], m=3, S_o=100 [g/l], k=2 [–], V_{max}=1.5 [l] [9, 11]. With (E) is noted the ethanol concentration.

Let **Z** be the set of alternatives (**Z**={specific growth rates-μ}=[0, 0.6]) and **P** be a convex subset of discrete probability distributions over **Z**. The expert "preference" relation over **P** is expressed through ($\succ$) and this is also true for those over **Z** (**Z**$\subseteq$**P**). As mentioned above the utility function is defined with precision up to affine transformation (interval scale). A decision support system for subjective utility evaluations is built and used. The results are shown on Figs. 2 and 3. The utility function is approximated by a polynomial:

$$U(\mu) = \sum_{i=0}^{6} c_i \mu^i . \qquad (6)$$

We denote with $U(\mu)$ the DM expert function used in the control design. The polynomial representation permits exact analytical determination of the derivative of the utility function and determination of the optimal technological parameters, optimal specific growth rate (optimal set point) (Fig. 3) [8, 9]. The utility is evaluated with 64 learning points. This number of questions is for a primary orientation**.**

The seesaw line in Fig. 4 is pattern recognition of $\mathbf{A_u}$ and $\mathbf{B_u}$.

This seesaw line recognizes correctly more then 97% of the expert answers. The polynomial approximation of the DM utility function $U(\mu)$ is the smooth line in Fig. 5 (the mathematical expectation). The utility function is determined with precision up to affine transformation.

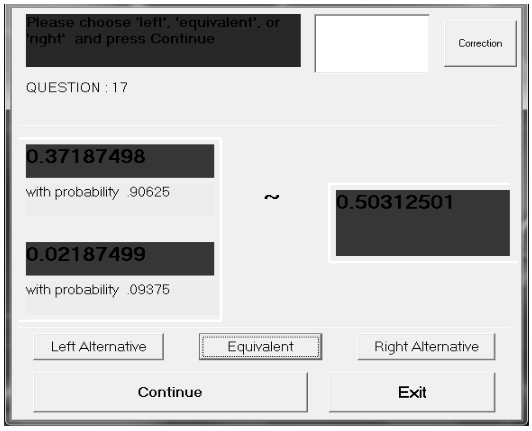

Fig. 2. Decision support system.

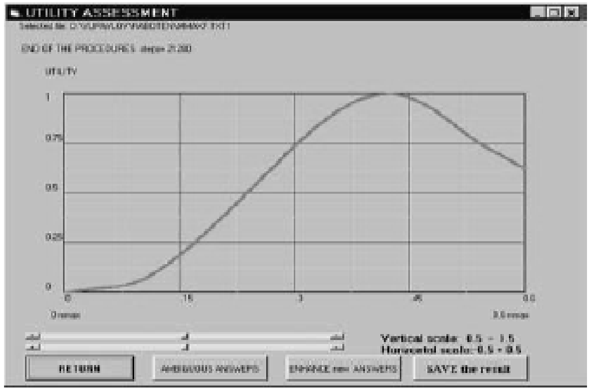

Fig. 3. Utility function evaluation.

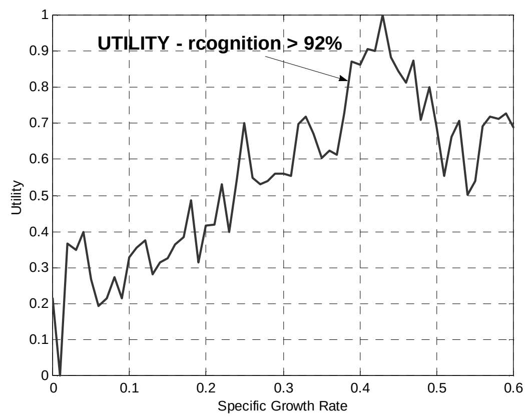

Fig. 4. Pattern recognition.

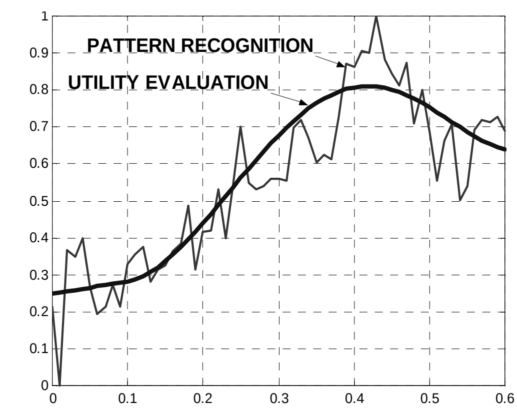

Fig. 5. Utility evaluation of the Specific Growth Rate.

The DM’s utility recognizes correctly more then 81% of the expert answers (learning points and DM’s answers). The maximum of the utility function determines the “best”, the optimal set point of the fed-batch cultivation process after the technologist [8].

4. CONTROL DESIGN AND STABILIZATION OF A COMPLEX CULTIVATION PROCESS

We preserve the notation $U(.)$ for the DM utility used in the control design. The control design is based on the solution of the next optimal control problem:

Max($U(\mu)$) for minimal time, *where the variable μ is the specific growth rate, ($\mu\in[0, \mu_{max}]$). Here $U(\mu)$ is an aggregation objective function (the utility function)* and *D is the control input* (*the dilution rate $D\in[0, D_{max}]$*):

$$\begin{aligned}
&\max(U(\mu)),\ \mu\in[0,\mu_{\max}],\ t\in[0,T_{\text{int}}],\ D\in[0,D_{\max}],\\
&\dot{X}=\mu X-DX,\\
&\dot{S}=-k\mu X+(So-S)D,\\
&\dot{\mu}=m\left(\mu_m\frac{S}{(K_S+S)}-\mu\right).
\end{aligned} \tag{7}$$

The differential equation in (7) describes *a continuous fermentation process*. The Monod-Wang model permits exact linearization to Brunovsky normal form [1, 3, 9, and 12]. The optimal solution is determined with the use of the Brunovsky normal form of model (7):

$$\begin{aligned}
\dot{Y}_1&=Y_2,\\
\dot{Y}_2&=Y_3,\\
\dot{Y}_3&=W.
\end{aligned} \tag{8}$$

In the formula above, W denotes the control input of the Brunovsky model (8). The vector (Y_1, Y_2, Y_3) is the new state vector [9, 11]:

$$\begin{aligned}
&Y_1=u_1,\\
&Y_2=u_3(u_1-ku_1^2),\\
&Y_3=u_3^2(u_1-3ku_1^2+2k^2u_1^3)+m\left(\mu_m\frac{u_2}{(K_S+u_2)}-u_3\right)(u_1-ku_1^2).
\end{aligned}$$

$$\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\Phi(X,S,\mu)=\begin{pmatrix}\dfrac{X}{S_0-S}\\ S\\ \mu\end{pmatrix}. \tag{9}$$

The derivative of the function Y_3 determines the interconnection between W and D. The control design is a design based on the Brunovsky normal form and application of the Pontrjagin’s maximum principle step by step for sufficiently small time periods T [4, 7, 9, and 11]. The interval T could be the step of discretization of the differential equation solver. The optimal control law has the analytical form:

$$D_{opt}=\mathbf{sign}\left(\left(\sum_{i=1}^{6}ic_i\mu^{(i-1)}\right)(T-t)\left[\frac{(T-t)\mu(1-2kY_1)}{2}-1\right]\right)D_{\max},$$
where (10)
$$\mathbf{sign}(r)=1, r>0,\ \mathbf{sign}(r)=0, r\le 0.$$

The sum is the derivative of the utility function. It is clear that the optimal “time-minimization” control is determined from the sign of the utility derivative. The control input is $D=D_{\max}$ or $D=0$. The solution is in fact a ***“****time-minimization****”*** control (if the time period T_{int} is sufficiently small). The control brings the system back to the set point for minimal time in any case of specific growth rate deviations [9 and 11].

The control law *of the fed-batch process* has the same form because $D(t)$ is replaced with $F(t)/V(t)$ in Monod-Wang model (5). Thus, the feeding rate $F(t)$ takes $F(t)=F_{\max}$ or $F(t)=0$, depending on $D(t)$ which takes $D=D_{\max}$ or $D=0$.

We conclude that the control law (10) brings the system to the set point (optimal growth rate) with a ”time minimization” control, starting from any deviation of the specific growth rate (Fig. 6).

Thus, we design the next control law:

- Time interval − [0, t_1]: the control is a **“**time-minimization**”** control − formula (10), where $\mu(t_1)=(x_{30}-\varepsilon)$**,** $\varepsilon>0$, x_{30} is determined by the max**(**$U(\mu)$) and ε is a sufficiently small value. The input D is replaced with $F=F_{\max}$, when $D=D_{\max}$;
- Time interval − [t_1, t_2]: the control law is $F=0$ ($\mu(t_1)=(x_{30}-\varepsilon)$**,** $\mu(t_2)=\mathbf{x_{30}}$ and $d/dt(\mu(t_2))=0$ (to be avoided the over-regulation shown on Fig. 6);
- After the moment t_2 the control is again the control (10) with $F=\gamma F_{\max}$, when $D=D_{\max}$ (chattering control with $1 \ge \gamma > 0$). The choice of γ depends on the step of the equation solver and on bioreactor characteristics and is not a part of the optimization (in this investigation $\gamma = 0.123$);

The deviations of the fed-batch process with this control law are shown on Figs. 6 and 7.

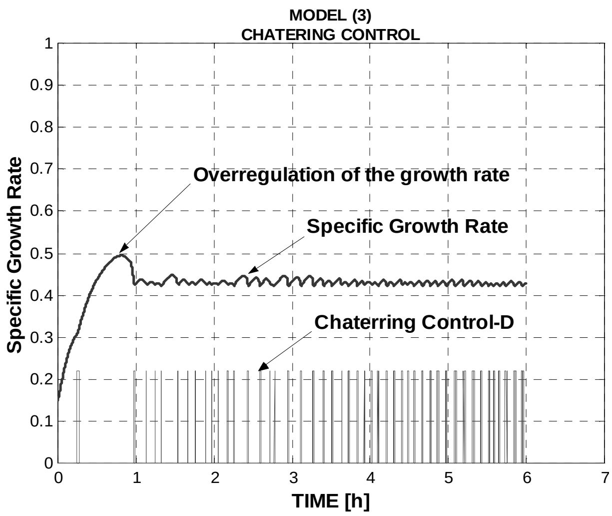

Fig. 6. Chattering Growth rate control.

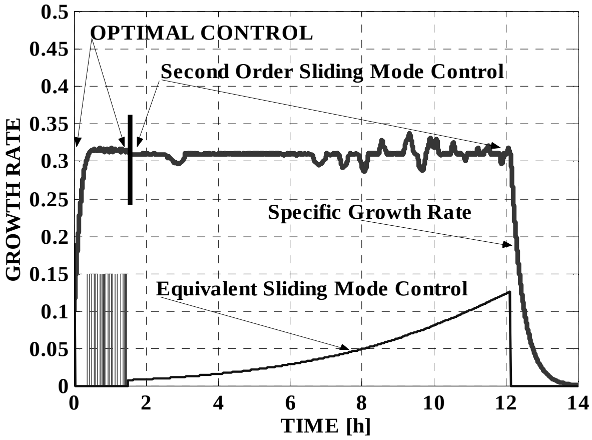

Fig. 7. Stabilization of the fed-batch process.

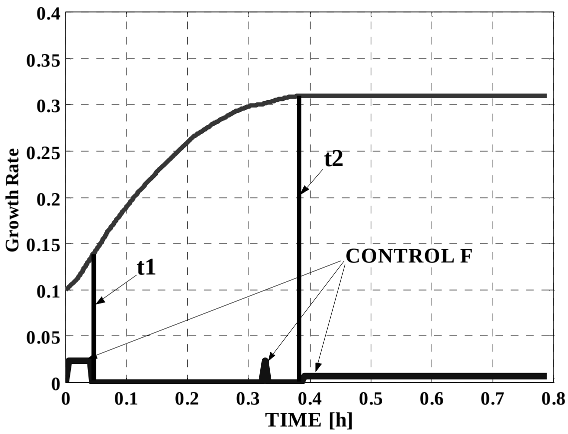

Fig. 8. Optimal growth rate profile.

After the stabilization in the "best" growth rate position the system can be maintained around the optimal parameters with a sliding mode control (Fig. 7) [9, 11, and 15].

The most difficult part of this investigation is determination of approximations of moment t_1 and moment t_2 [9 and 11]. This solution is shown on Fig. 8. The determination of t_1 needs resolution of a transcendent equation. We propose an approximation of moment t_1. This approximation is determined by the moment when the vector state of the system across a manifold, approximation of the exact solution [9 and 11]. This approximation could be iteratively repeated until the system attains the nominal technological position smoothly (Fig. 8).

The Monod-Wang kinetic model is a partial case of a more complex Monod-Wang-Yerusalimsky kinetic one:

$$\begin{aligned}
\dot{X} &= \mu X - \frac{F}{V}X, \\
\dot{S} &= -k\mu X + (So - S)\frac{F}{V}, \\
\dot{\mu} &= m\left(\mu_m \frac{S}{(K_S + S)}\frac{k_E}{(k_E + X)} - \mu\right), \\
\dot{V} &= F, \\
\dot{E} &= k_2\,\mu E - \frac{F}{V}E.
\end{aligned} \qquad (11)$$

In this formula, k_E is a constant. The 5th equation describes the production of ethanol (E). The equivalent model of the continuous Monod-Wang-Yerusalimsky model has the form [11]:

$$\begin{aligned}
\dot{Y}_1 &= Y_2, \\
\dot{Y}_2 &= Y_3, \\
\dot{Y}_3 &= W, \\
\dot{Y}_4 &= Y_4 Y_2 \frac{(k_2 - kY_1)}{Y_1(1 - kY_1)}.
\end{aligned} \qquad (12)$$

The control input of is W. The state vector has the following explicit form:

$$\begin{aligned}
Y_1 &= u_1 \\
Y_2 &= u_3(u_1 - ku_1^2) \\
Y_3 &= u_3^2\,(u_1 - 3ku_1^2 + 2k^2u_1^3) + m(u_1 - ku_1^2)\times \\
&\quad \times\left(\mu_m \frac{u_2 k_E}{(K_S + u_2)(k_E + u_1(So - u_2))} - u_3\right) \\
Y_4 &= u_4.
\end{aligned} \qquad (13)$$

The last equation in formula (12) could be transformed to the following form:

$$\begin{aligned}
\dot{Y}_4 &= Y_4 Y_2 \frac{(k_2 - kY_1)}{Y_1(1 - kY_1)} = Y_4 \dot{Y}_1 \frac{(k_2 - kY_1)}{Y_1(1 - kY_1)}, \\
\frac{\dot{Y}_4}{Y_4} &= \dot{Y}_1 \frac{(k_2 - kY_1)}{Y_1(1 - kY_1)}.
\end{aligned} \qquad (14)$$

Consecutively the variable Y_4 depends only on Y_1. The solution of this equation is the following:

$$Y_4 = k_4 Y_1^{k_2}\left|1 - kY_1\right|^{(1 - k_2)}, \quad k_4 \in \boldsymbol{R}. \qquad (15)$$

This solution shows that model (12) is dynamically equivalent to the Brunovsky normal form described by formula (8) [11]:

$$\begin{aligned}
\dot{Y}_1 &= Y_2, \\
\dot{Y}_2 &= Y_3, \\
\dot{Y}_3 &= W.
\end{aligned} \qquad (16)$$

The Monod-Wang-Yerusalimsky kinetic model has the same Brunovsky normal form as this of Monod-Wang kinetic model. That is why we could apply the same mathematical technique. The control solution is the same, the optimal profile is the same and the control law is the same, but with a small difference. The approximation of moment t_1 needs a more complex solution.

The Monod-Wang-Yrusalimsky kinetic model could be applied in the functional states with distinctive occurrence of an acetate inhibition effect.

5. CONCLUSIONS

In the paper a mathematical utility evaluation procedure for elimination of the uncertainty in the decision-maker's preferences is proposed. The approach permits iterative and precise evaluation of the "best" specific growth rate of the fed-batch process and iterative control design in agreement with the DM's preferences as maximum of this utility function.

An example of complex control design based on the evaluated DM's utility function is demonstrated considering an *Escherichia coli* fed-batch cultivation process. Considering the *Escherichia coli* fed-batch process the mathematical descriptions for the different functional states is based on Monod and Yerusalimsky kinetic models. That is why Monod -Wang and Monod-Wang-Yrusalimsky kinetic models could describe completely this fed-batch cultivation by a sequence of successive utilization of these models. The parameters of the models will change in the different functional sates. The Monod-Wang-Yrusalimsky kinetic model could be applied in the functional states with occurrence of an acetate inhibition effect.

The stochastic utility evaluation approach and the designed decision support system could be used in optimization procedures and control design of complex processes and for description of the complex system "technologist- dynamic model".

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