Proceedings in Manufacturing Systems, Volume 6, Issue 4, 2011 ISSN 2067-9238

OPTIMUM DESIGN POSSIBILITIES OF THE FOOT SOLES SHAPE AND A METHOD OF REDUCING THE CONSUMED ENERGY BY THE MODULAR WALKING ROBOT

Ion ION[1,*], Ion SIMIONESCU[2], Aurelian VASILE[3], Dan SIRMAN[4]

[1)] PhD, Assoc. Prof., Technology of Manufacturing Department, University "Politehnica" of Bucharest, Romania
[2)] PhD, Prof., Mechanism and Robot Theory Department, University "Politehnica" of Bucharest, Romania
[3)] PhD Student, Mechanism and Robot Theory Department, University "Politehnica" of Bucharest, Romania
[4)] PhD Student, Mechanism and Robot Theory Department, University "Politehnica" of Bucharest, Romania

Abstract: *Distribution of reaction forces from the supports of the legs is one of the important problems that must be resolved to organize movements on land legs walking robots with relief complicated. Some robot foot soles have curved front and rear ends, when the foot soles land on or are lifted off the terrain, they are in linear contact with the terrain. Feet walking robots in general, particularly those bipedal must be such that they can move with a uniform speed and also quickly, similar to human walking. If feet are not appropriate forms which adapt to uneven terrain, they are not able to apply the necessary driving forces of land movement, resulting in a high enough reaction force from the land. The paper relates some particularities and the optimum design possibilities of the foot soles shape and presents a method of reducing the consumed energy by the walking robot while shifting.*

Key words: *foot sole, gait, leg, modular walking robot, static balancing mechanisms.*

1. INTRODUCTION

The modular walking robots [MERO] represent a special category of robots, characterized by having the power source and technological equipments embarked on the platform which can be transported on not arranged, horizontal and rough terrain.

Platform's weight force is distributed on each leg in the support phase at that particular moment, depending on the platform's weight centre position.

For this reason there is an obvious dependence between the robot's energy consumption and it's shifting autonomy.

Energetic consumption of the walking robot can be seen as made of two main parts:

- the mechanical work the robot needs for shifting ;
- the mechanical work required for sustaining the platform at a given height in the gravitational field.

Both the optimization of the legs elements cinematic dimensions and designing of the static balancing systems are done under the hypothesis that the robot shifts on an even horizontal surface, without obstacles, at a steady speed and taking into account a certain form of leg's support point trajectory, in comparison with the platform.

Considering this division, the drop in the energy consumption consists in designing the legs mechanisms optimally, the objective function being the one which expresses the mechanical work done for shifting the robot by a step.

[*] Corresponding author: Splaiul Independentei 313, sector 6, Bucharest, 060042, Romania
Tel:0214029356
Fax: 0214029373
E-mail:*ioni51@yahoo.com* (I. Ion)

Another possibility consists in balancing the platform's and legs elements gravitational forces using static balancing systems. These systems are attached to the legs mechanisms and take action only in the support phases of the legs.

Only under such circumstances the energy consumption is minimum.

In the case the modular walking robot is shifting at a variable speed, on a surface which is not even and horizontal, which contains obstacles, the energy consumption has a larger value.

Even in this case energy saving is also achieved, by the effects of the optimum dimensions of the legs mechanisms and of the static balancing systems are smaller.

Considering the fact that the energy source is fixed on the robot's platform, the dimensions of the legs mechanisms' elements are calculated using a multicritical optimization proceeding, which includes several restrictions. The objective function may express:

- the mechanical work needed for shifting the platform by one step ;
- maximum moving force for the leg mechanism;
- the maximum power required for shifting .

These objective functions can be considered separately or simultaneously.

2. FEET PROBLEMS OF ROBOTS WALKING

At the "Politehnica" University of Bucharest several models of MERO (MEchanism RObot) modular walking robots were developed [4, 6, 7, and 8].

These walking robots have three or more modules (bipedal walking robots) Figs. 3 and 4. Each leg of the bipedal walking robots has three degrees o freedom.

Biped robot module is part of the modular walking robots.

Feet walking robots in general, particularly those bipezi must be such that they can move with a uniform and went quickly, similar to human walking.

Land moving biped robot may not be flat and horizontal, and may have irregularities.

Biped robot module is desirable to have suitable feet walking on any surface.

If feet are not appropriate forms which adapt to uneven terrain, they are not able to apply the necessary driving forces of land movement, resulting in a high enough reaction force from the land.

It is impossible to precisely control the position and direction of advance, the robot is false steps on the ground and prevents.

When foot bipedal modules of walking robots are placed on the ground, she suffers a shock more or less powerful because the ground reaction force. The shock damping is required in order to transmit the necessary forces to land in order to maintain the desired position of the robot body and to achieve its verticality control consistent regular stepping movements of legs.

Some bipezi module walking robots have curved feet in front of corresponding 'fingertips, and in the back for the "heel".

But curves feet from the front and rear, make bipedal robot to be unstable when kept at rest and standing on his feet, since it reduces the surface of the sole of each foot, which is in contact with the ground.

When such a foot base sits on land or on land is high, there is a theoretical linear foot contact with the ground. If the land surface is horizontal, touch base with the ground is unstable.

Even if the ground is horizontal, actual contact with the ground pad is a rectangular area around a very narrow width foot long foot.

As a result, the foot rotation occurs around a vertical axis, since there is cause instability and foot can not produce a force strong enough to oppose big time.

Walking robots can therefore deviate from the direction set to advance and move in an unexpected direction.

The stability of a bipedal module of the walking robots at rest is very important.

While a bipedal module of the walking robots is at rest, is not desirable − in terms of energy savings − as drive motors are supplied with energy, so always be kept locked.

But an interruption of power supply to engine damaging static stability of the robot.

Some problems of stability during bipedal walking robots stepping movement are further analyzed.

Modular walking robots stability of quadruped constructive variant is much better than the stability of biped robot module, because the ratio between the size of the support polygon [8 and 10] and height of center of gravity of the robot in its entirety is much higher than in the biped robot quadruped.

Moreover, there are unavoidable period of time that bipedal robot is based on a single leg during stepping.

One can imagine a limit situation in which a biped robot runs, there are time periods when both feet are in the transfer phase.

Control system operation stability of bipedal walking robots may include a visual sensor, a sensor equalization and necessarily, sensors for measuring the angles between adjacent elements of kinematic.

Feet associated sensors must be able to detect the shape of an object that is trampled or irregularities land as bipedal robot is obliged to step into the same space as man and there are many similar objects with a step threshold, a cylindrical rod or other objects irregular surfaces that can seat base.

Foot pad structure of walking robots in general, a bipedal module of walking robots in particular, must adapt to small surface irregularities on the ground moving.

This structure allows the reaction forces to be transmitted smoothly between feet and ground, so the robot can be controlled the position and direction of travel.

Foot has one cylindrical surface, arranged at each end of it.

So this area should form such that bipedal movements to make the acceleration as small forces or moments are needed as low propulsion engines.

Modular walking robots equipped with so feet can walk evenly and quickly, and - also - with an increased efficiency of energy transfer phase leg, in a way similar to human walking.

It is preferable that the generators are facing curved surfaces in the transverse direction, normal direction of advance.

With this arrangement, stable walking robots can walk even when the foot is placed in different positions on the ground. Foot structure may include an elastic layer applied over the surface of the base curve.

This layer absorbs the shocks that occur when base is placed on the ground, resulting in went very smoothly. Elastic layer must first be deformed in proportion to applied load, then deflection to be nonlinear.

These two features elastic layer absorbs shocks arising best foot to put on the ground and allow the base to be able to adapt well to any kind of land area to achieve a stable gait.

Foot structure can include at least one projection located at one end of the foot (front or back). Projections are deformable or rigid and mobile, they can move inside and outside the base.

This solution design compensates to some extent reduce the static stability of the robot due to curvature "peak" and "heel", but allows him to walk evenly and have a stable position when at rest.

The role of these projections is to create a support contour with the largest possible area.

Projection can be made from an elastic material that deforms under the action tasks, or a rigid material, in which case it is retractable to the lower surface of the base when pressed to the ground reaction force.

These projections also help cushion to absorb shocks that occur when the foot is resting on the floor to achieve a stable gait.

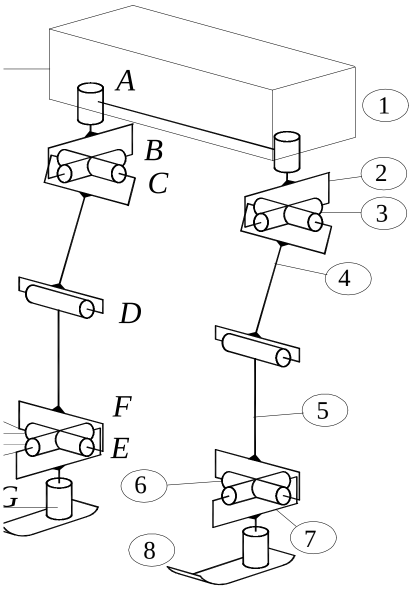

Fig. 1. Scheme of the biped robot module part of the modular walking robots.

Foot pad structure includes projections require additional springs designed to hold the position, which is retractable by compression springs, in response to increased loading.

Foot may have more projections aligned in the transverse direction, normal direction of advance. In another constructive option, projections are driven from power system of the robot drive means being such as to move projections inside and outside "peak" and "heel".

Such a robot can walk faster with a uniform and efficient use of energy, and when at rest, maintain a stable position.

In Fig. 1 is outlined a bipedal module of walking robots [4], whose legs are formed each a "thigh" (4), a "shank" (5) and a "foot" (8), connected to the body (1). The two legs are identical and symmetrical body axis (1). "Laba" (8) of the foot is connected to the "leg" (5) through a kinematic chain composed of elements (6) and (7), connected by rotating couplers E, F and G with two two perpendicular axes.

This kinematic chain ankle is simulating human foot. Similarly, the "thigh" (4) is connected to the body (1) the kinematic chain composed of elements (2) and (3) and joints A, B and C, which simulates human leg hip joint. "Thigh" is linked to the "leg" through joint kinematics D.

All seven joints of each foot is leading. Leg mechanism, the general shape space (Fig. 1), has seven degrees of the transfer phase and support phase, when ground contact is the "tip" or "heel" feet.

When foot rests on the soil surface, the mechanism has only one leg mobility. On the straight, kinematic couplings A, B, F and G, whose axes are contained the plane in which the body moves (1), "thigh" (4) "calf" (5) and "foot" (8) are operated leg.

Therefore, in this particular case, the mechanism leg is flat and has only three degrees of phase transfer.

Leg movement's elements necessary for making stepping can be done and where the base surface is flat.

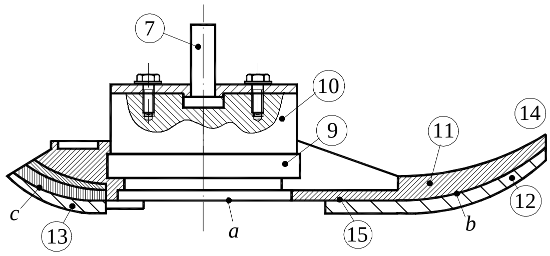

Fig. 2. The cross section through foot [4, 9].

But command and control the relative movements of the three components of each leg of the biped robot is facilitated by the use of human-like feet, which have ends, front and rear, made in the form of convex cylindrical surfaces (Fig.2).

With these shoes, bipedal robot moving uniformly, without changes in forward speed sensitive.

Cylindrical shapes of these surfaces is such that moments of maximum engine driving couplers, phase support is as small and movement is as uniform, with variations as low speeds.

Figure 2 presents a cross section through the "foot" foot, made with a vertical and parallel to the direction of advance.

Foot has compose a force sensor (9) six axes, placed under the rotating element (10) for measuring three-dimensional components Fx, Fy and Fz of the reaction force components Mx, My, Mz of the moment reaction.

By foot it is determined whether or not in contact with the ground and if it is in contact, which is the size and direction of loading.

Foot has a frame (11) shaped boat, made of a rigid material, resistant and easily positioned under the force sensor (9).

Framework (11) has a central flat bottom surface (a), a corresponding portion of the fingertips (b) located at the front end of the base and a "heel" (c) located at the rear.

Elastic layers (12) and (13), made of rubber or other elastic material with appropriate stiffness, are glued to "tip" (b) and the "heel" (c) and are designed to absorb or cushion the shock produced to put foot on land, and allow the foot to adapt to the terrain surface irregularities.

Elastic layers (12) and (13) cover portions of three regions corresponding to the base support leg.

3. ANTHROPOMORPHIC FORMS FLAT FEET OPTIMIZATION OF MODULAR WALKING ROBOTS

Legs walking robots generally of the bipedal in particular, must be such that they can move with a gait smooth and quick, similar to human walking.

The current conventional bipedal robot is rather poor and cannot move evenly and quickly in all conditions, like a man.

The feet of the walking robots in general, the modular walking robots especially (Figs. 3 and 4) [4 and 5] must be build so that the robots are able to move with smooth and quick gait [4].

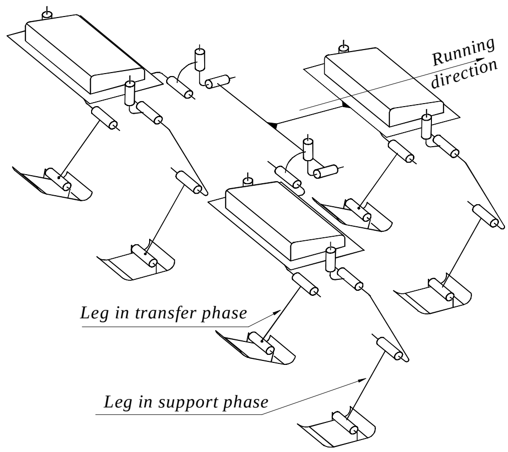

Fig. 3. Scheme of the modular walking robot MERO [4].

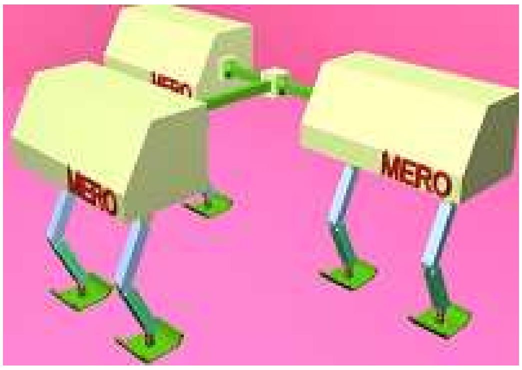

Fig. 4. MERO modular walking robot computer graphics [6, 7].

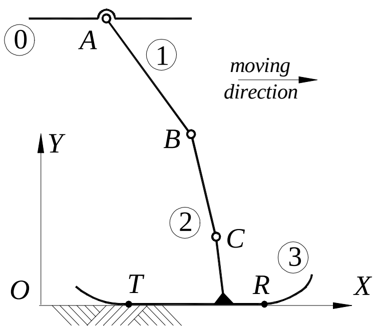

Fig. 5. Kinematics scheme of the an anthropomorphous leg.

In a simplified form, the leg of a modular walking robot is build by three members [4] (Fig. 5), namely thigh (1), shank (2) and foot (3). All of the joint axes are parallel with the support plane of the land.

The modular robot foot soles have curved front and rear ends, corresponding to the toes tip and to the heel respectively.

If the position of the axis of the pair A is defined with respect to the fixed coordinate axes system fastened on the support plane, the leg mechanism has a degree of freedom in the support phase and three degree of freedom in the transfer phase.

Therefore, the angles φ_1 and φ_2 (Fig. 6) and the distance S, which defines the positions of the leg elements, cannot be calculated only in term of the coordinates X_A, Y_A.

In other words, an unknown must be specified irrespective of the coordinates of the center of the pair A. In consequence, the foot (3) always may step on the land with the flat surface of the sole.

The module body may be moved with respect to the terrain without the changing of foot (3) position.

This walking possibility is not similarly with human walking and may be achieved only if the velocity and acceleration of the robot body is small.

In general, the foot can be support on the land both with the flat surface and the curved front and rear ends.

The plane surface of the sole and the cylindrical surface of the front end are tangent along of the generatrix R (Fig. 5).

In the plane of motion, the position of the generatrix R with respect to the mobile coordinate axes system is given by the coordinate's x_{3R}, y_{3R}.

The size of the flat surface of the foot, i.e. the position of the generatrix R, is determined in terms of allowable pressure on the terrain.

The curve directories of the cylindrical surface of the front end is defined by the parametrical equations: $x_3 = x_3(\lambda)$, $y_3 = y_3(\lambda)$, with respect to the mobile coordinate axes system attached to this element.

The generatrix in which the plane surface of the foot is tangent with the cylindrical surface of the front end is positioned by the parameter λ_0: $x_{3R} = x_3(\lambda_0)$, $y_{3R} = y_3(\lambda_0)$.

3.1. Kinematics Analysis of the Leg Mechanism

In the support phase, when the flat surface of the foot is in contact with the terrain (Fig. 6.*a*), the analysis equations are:

$$X_A + AB\cos\varphi_1 + BC\cos\varphi_2 - S = 0;$$

$$Y_A + AB\sin\varphi_1 + BC\sin\varphi_2 - x_{3R} = 0. \qquad (1)$$

The system is indeterminate because contains three unknowns, namely φ_1, φ_2 and S. In order to solve it, the value of an unknown must be imposed, for example the angle φ_1. It is considered as known the position of the pair axis A. In this hypothesis, the solutions of the system (1) are:

$$\varphi_2 = \arccos\frac{\sqrt{BC^2 - (x_{3R} - Y_A - AB\sin\varphi_1)^2}}{BC};$$

$$\varphi_2 = \arcsin\frac{x_{3R} - Y_A - AB\sin\varphi_1}{BC};$$

$$S = \sqrt{BC^2 - (x_{3R} - Y_A - AB\sin\varphi_1)^2}.$$

The coordinates of the tangent point R have the expressions:

$$X_R = X_A + \sqrt{BC^2 - (x_{3R} - Y_A - AB\sin\varphi_1)^2} + y_{3R}.$$

$$Y_R = 0.$$

In the end of the support phase, when the contact of the foot with the land is made along the generatrix which passes through the P point (Fig. 6.*b*), the analysis equations are:

$$Y_A + AB\sin\varphi_1 + BC\sin\varphi_2 + CP\sin(\varphi_3 + u) = Y_P;$$

$X_A + AB\cos\varphi_1 + BC\cos\varphi_2 + CP\cos(\varphi_3 + u) = X_P;$

$$\frac{\left.\dfrac{\mathrm{d}x_3}{\mathrm{d}\lambda}\right|_{\lambda=\lambda_P}}{\left.\dfrac{dy_3}{d\lambda}\right|_{\lambda=\lambda_P}} = -\tan\varphi_3\,, \qquad (2)$$

where : $u = \arctan\dfrac{y_3(\lambda_P)}{x_3(\lambda_P)}$; $CP = \sqrt{x_3^2(\lambda_P) + y_3^2(\lambda_P)}$;

λ_P is the value of the parameter λ which corresponding to the generator passing through the point P of curve directories;

$$X_P = X_R + \int_{\lambda_0}^{\lambda_P}\sqrt{\left(\frac{\mathrm{d}x_3}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}y_3}{\mathrm{d}\lambda}\right)^2}\,\mathrm{d}\lambda\,,$$

because it is assumed that the foot sole do not slipped on the land surface.

The equations (2) are solved with respect to the unknowns φ_2, φ_3 and λ_P. in terms of the coordinates X_A, Y_A of the center of the couple A and the and the amount imposed angle φ_1.

By differentiation with respect to the time of the equations (2), the velocity transmission functions result:

$$\frac{\mathrm{d}Y_A}{\mathrm{d}t} + AB\cos\varphi_1\frac{\mathrm{d}\varphi_1}{\mathrm{d}t} + BC\cos\varphi_2\frac{\mathrm{d}\varphi_2}{\mathrm{d}t} + CP$$

$$\cos(\varphi_3+u)\left(\frac{\mathrm{d}\varphi_3}{\mathrm{d}t} + \frac{\mathrm{d}u}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}t}\right) + \frac{\mathrm{d}CP}{\mathrm{d}\lambda}\sin(\varphi_3+u)\frac{\mathrm{d}\lambda}{\mathrm{d}t} = 0;$$

$$AB\sin\varphi_1\frac{\mathrm{d}\varphi_1}{\mathrm{d}t} + BC\sin\varphi_2\frac{\mathrm{d}\varphi_2}{\mathrm{d}t} + CP\sin(\varphi_3+u)$$

$$\left(\frac{\mathrm{d}\varphi_3}{\mathrm{d}t} + \frac{\mathrm{d}u}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}t}\right) - \frac{\mathrm{d}CP}{\mathrm{d}\lambda}\cos(\varphi_3+u)\frac{\mathrm{d}\lambda}{\mathrm{d}t} + \frac{\mathrm{d}X_P}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}t} -$$

$$\frac{\mathrm{d}X_A}{\mathrm{d}t} = 0; \qquad (3)$$

$$\frac{\dfrac{\mathrm{d}^2y_3}{\mathrm{d}\lambda^2}\dfrac{\mathrm{d}x_3}{\mathrm{d}\lambda} - \dfrac{\mathrm{d}^2x_3}{\mathrm{d}\lambda^2}\dfrac{\mathrm{d}y_3}{\mathrm{d}\lambda}}{\left(\dfrac{\mathrm{d}x_3}{\mathrm{d}\lambda}\right)^2}\frac{\mathrm{d}\lambda}{\mathrm{d}t} + \frac{1}{\cos^2\varphi_3}\frac{\mathrm{d}\varphi_3}{\mathrm{d}t} = 0$$

which are simultaneous solved with respect to the unknowns $\dfrac{\mathrm{d}\varphi_2}{\mathrm{d}t}$, $\dfrac{\mathrm{d}\varphi_3}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}\lambda}{\mathrm{d}t}$, where:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\dfrac{\mathrm{d}y_3}{\mathrm{d}\lambda}x_3(\lambda) - \dfrac{\mathrm{d}x_3}{\mathrm{d}\lambda}y_3(\lambda)}{x_3^2(\lambda) + y_3^2(\lambda)}\frac{\mathrm{d}\lambda}{\mathrm{d}t};$$

$$\frac{\mathrm{d}CP}{\mathrm{d}t} = \frac{x_2(\lambda)\dfrac{\mathrm{d}x_2}{\mathrm{d}\lambda} + y_2(\lambda)\dfrac{\mathrm{d}y_2}{\mathrm{d}\lambda}}{CP}\frac{\mathrm{d}\lambda}{\mathrm{d}t};$$

$$\frac{\mathrm{d}X_P}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left[\int_{\lambda_0}^{\lambda}\sqrt{\left(\frac{\mathrm{d}x_3}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}y_3}{\mathrm{d}\lambda}\right)^2}\,\mathrm{d}\lambda\right]\frac{\mathrm{d}\lambda}{\mathrm{d}t}.$$

Further on, by differentiation the equations (3) result the acceleration transmission functions:

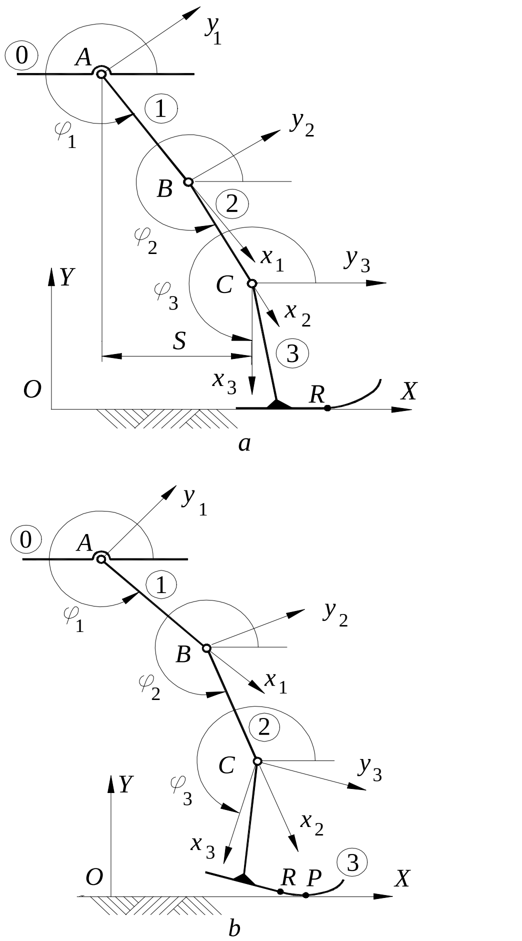

Fig. 6. The leg in the support phase.

$$\frac{\mathrm{d}^2Y_A}{\mathrm{d}t^2} + AB\left[\cos\varphi_1\frac{\mathrm{d}^2\varphi_1}{\mathrm{d}t^2} - \sin\varphi_1\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}t}\right)^2\right] +$$

$$BC\left[\cos\varphi_2\frac{\mathrm{d}^2\varphi_2}{\mathrm{d}t^2} - \sin\varphi_2\left(\frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right)^2\right] +$$

$$CP\cos(\varphi_3+u)\left(\frac{\mathrm{d}^2\varphi_3}{\mathrm{d}t^2} + \frac{\mathrm{d}^2u}{\mathrm{d}\lambda^2}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}t}\right)^2 + \frac{\mathrm{d}u}{\mathrm{d}\lambda}\frac{\mathrm{d}^2\lambda}{\mathrm{d}t^2}\right) -$$

$$CP\sin(\varphi_3+u)\left(\frac{\mathrm{d}\varphi_3}{\mathrm{d}t} + \frac{\mathrm{d}u}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}t}\right)^2 +$$

$$+\frac{\mathrm{d}^2CP}{\mathrm{d}\lambda^2}\sin(\varphi_3+u)\left(\frac{\mathrm{d}\lambda}{\mathrm{d}t}\right)^2 + 2\frac{\mathrm{d}CP}{\mathrm{d}\lambda}\cos(\varphi_3+u)$$

$$\left(\frac{\mathrm{d}\varphi_3}{\mathrm{d}t} + \frac{\mathrm{d}u}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}t}\right)\frac{\mathrm{d}\lambda}{\mathrm{d}t} + \frac{\mathrm{d}CP}{\mathrm{d}\lambda}\sin(\varphi_3+u)\frac{\mathrm{d}^2\lambda}{\mathrm{d}t^2} = 0;$$

$$AB\left[\sin\varphi_1\frac{\mathrm{d}^2\varphi_1}{\mathrm{d}t^2} + \cos\varphi_1\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}t}\right)^2\right] + BC\left[\sin\varphi_2\frac{\mathrm{d}^2\varphi_2}{\mathrm{d}t^2} + \right.$$

$$\left.\cos\varphi_2\left(\frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right)^2\right] + CP\sin(\varphi_3+\mathrm{u})$$

$$\left.\mathrm{s}\varphi_2\left(\frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right)^2\right] + CP\sin(\varphi_3+\mathrm{u})$$

$$\left(\frac{d^2\varphi_3}{dt^2}+\frac{d^2u}{d\lambda^2}\left(\frac{d\lambda}{dt}\right)^2+\frac{du}{d\lambda}\frac{d^2\lambda}{dt^2}\right)+CP\cos(\varphi_3+u)$$

$$\left(\frac{d\varphi_3}{dt}+\frac{du}{d\lambda}\frac{d\lambda}{dt}\right)^2+\frac{d^2X_P}{d\lambda^2}\left(\frac{d\lambda}{dt}\right)^2+$$

$$+2\frac{dCP}{d\lambda}\sin(\varphi_3+u)\left(\frac{d\varphi_3}{dt}+\frac{du}{d\lambda}\frac{d\lambda}{dt}\right)\frac{d\lambda}{dt}-\frac{d^2X_A}{dt^2}-$$

$$\frac{d^2CP}{d\lambda^2}\cos(\varphi_3+u)\left(\frac{d\lambda}{dt}\right)^2-\frac{dCP}{d\lambda}\cos(\varphi_3+u)\frac{d^2\lambda}{dt^2}=$$

$$0;$$

$$\frac{\left(\frac{d^3y_3}{d\lambda^3}\frac{dx_3}{d\lambda}-\frac{d^3x_3}{d\lambda^3}\frac{dy_3}{d\lambda}\right)\frac{dx_3}{d\lambda}-2A\frac{d^2x_3}{d\lambda^2}}{\left(\frac{dx_3}{d\lambda}\right)^3}\left(\frac{d\lambda}{dt}\right)^2+\frac{A}{\left(\frac{dx_3}{d\lambda}\right)^2}\frac{d^2\lambda}{dt^2}+$$

$$\frac{2\sin\varphi_3}{\cos\varphi_3^3}\left(\frac{d\varphi_3}{dt}\right)^2+\frac{1}{\cos^2\varphi_3}\frac{d^2\varphi_3}{dt^2}=0,$$

where $A=\frac{d^2y_3}{d\lambda^2}\frac{dx_3}{d\lambda}-\frac{d^2x_3}{d\lambda^2}\frac{dy_3}{d\lambda}$,

which are simultaneous solved with respect to the unknowns $\frac{d^2\varphi_2}{dt^2}$, $\frac{d^2\varphi_3}{dt^2}$ and $\frac{d^2\lambda}{dt^2}$

where:

$$\frac{d^2u}{dt^2}=\left[\frac{\frac{d^2y_3}{d\lambda^2}x_3(\lambda)-\frac{d^2x_3}{d\lambda^2}y_3(\lambda)}{x_3^2(\lambda)+y_3^2(\lambda)}-\right.$$

$$\left.-2\frac{\frac{dx_3}{d\lambda}\frac{dy_3}{d\lambda}\left(x_3^2(\lambda)-y_3^2(\lambda)\right)-x_3(\lambda)y_3(\lambda)Q}{\left(x_3^2(\lambda)+y_3^2(\lambda)\right)^2}\right]\left(\frac{d\lambda}{dt}\right)^2+$$

$$+\frac{\frac{dy_3}{d\lambda}x_3(\lambda)-\frac{dx_3}{d\lambda}y_3(\lambda)}{x_3^2(\lambda)+y_3^2(\lambda)}\frac{d^2\lambda}{dt^2};\quad Q=\left(\frac{dx_3}{d\lambda}\right)^2-\left(\frac{dy_3}{d\lambda}\right)^2.$$

3.2. Forces Distribution in the Leg Mechanism

The goal of the forces analysis in the leg mechanism is the determination of the conditions of the static stability of the feet and of the fully robot [4, 6]. The leg mechanism is plane, and the reaction forces from the pairs are within the motion plane.

The pressure on the contact surface or generatrix is assumed to be equally distributed. From the equilibrium equations of the forces which act on the leg mechanism elements (Fig. 7), the reaction forces from pairs A, B and C and the modulus and the origin of the normal reaction N are calculated.

If the position of the origin of normal reaction force N is outside of the support surface, the foot overturns. To avoid this phenomenon it is enforce that the origin of the normal reaction force to fill a certain position, definite by the distance d. In this case, a driving moment M_{01} in the pair A, applied between the body (0) and the thigh (1) is added. This moment is the sixth unknown quantity of the forces distribution problem.

Taking into consideration the particularities of the contact between terrain and foot, the leg mechanism is analyzed in two steps:

- the first: it is solved the equations (4), which define the equilibrium of the forces acting on the elements (1) and (2),
- the second: it is solved the equations (5), which express the equilibrium of the forces acting on the foot (3).

The particularities consist in the fact that the foot (3) is supported or rolled without sliding on terrain.

As a result, the reaction force acting to the foot (3) has two components, namely $\overline{N}$ along the normal on the support plane and $\overline{T}$ holds in the support plane. The rolled without sliding of the front or rear end of the foot is done if $T < \mu N$ only, where μ is the frictional coefficient between foot and terrain.

The forces analysis is made in two situations.

1. The foot is supported with his flat surface on the terrain (Fig. 8a).

The equations of the forces equilibrium which act on the links (1) and (2) are:

$$Q_X+F_{i1X}-R_{12X}+R_{01X}=0;$$

$$Q_Y+F_{i1Y}-m_1g-R_{12Y}+R_{01Y}=0; \tag{4}$$

$$M_{01}+(F_{i1Y}-m_1g)(X_{G1}-X_B)-F_{i1X}(Y_{G1}-Y_B)+R_{01X}(Y_A-Y_B)+R_{01Y}(X_B-X_A)+M_{i1}=0;$$

$$F_{i2X}+R_{12X}+R_{32X}=0;\ F_{i2Y}-m_2g+R_{12Y}+R_{32Y}=0;$$

$$(F_{i2Y}-m_2g)(X_{G2}-X_B)-F_{12X}(Y_{G2}-Y_B)+R_{32Y}(X_C-X_B)-R_{32X}(Y_C-Y_B)+M_{i2}=0,$$

where: $M_{01}=0$.

$\overline{Q}=Q_x\overline{i}+Q_y\overline{j}$ is the direct acting load on the leg in the center of the pair A;

$$F_{ijX}=-m_j\frac{d^2X_{Gj}}{dt^2},F_{ijY}=-m_j\frac{d^2Y_{Gj}}{dt^2},\ M_{ij}=-I_{Gj}\frac{d^2\varphi_j}{dt^2};$$

$$j=\overline{1,3},$$

$$\frac{d^2X_{G1}}{dt^2}=\frac{d^2X_A}{dt^2}-(x_{1G1}\sin\varphi_1+y_{1G1}\cos\varphi_1)\frac{d^2\varphi_1}{dt^2}$$

$$-(x_{1G1}\cos\varphi_1-y_{1G1}\sin\varphi_1)\left(\frac{d\varphi_1}{dt}\right)^2;$$

$$\frac{d^2Y_{G1}}{dt^2}=\frac{d^2Y_A}{dt^2}+(x_{1G1}\cos\varphi_1-y_{1G1}\sin\varphi_1)\frac{d^2\varphi_1}{dt^2}$$

$$-(x_{1G1}\sin\varphi_1+y_{1G1}\cos\varphi_1)\left(\frac{d\varphi_1}{dt}\right)^2;$$

$$\frac{d^2X_{G2}}{dt^2}=\frac{d^2X_A}{dt^2}-(x_{2G2}\sin\varphi_2+y_{2G2}\cos\varphi_2)\frac{d^2\varphi_2}{dt^2}$$

$$-(x_{2G2}\cos\varphi_2-y_{2G2}\sin\varphi_2)\left(\frac{d\varphi_2}{dt}\right)^2-$$

$$-AB(\sin\varphi_1\frac{d^2\varphi_1}{dt^2}+\cos\varphi_1\left(\frac{d\varphi_1}{dt}\right)^2);$$

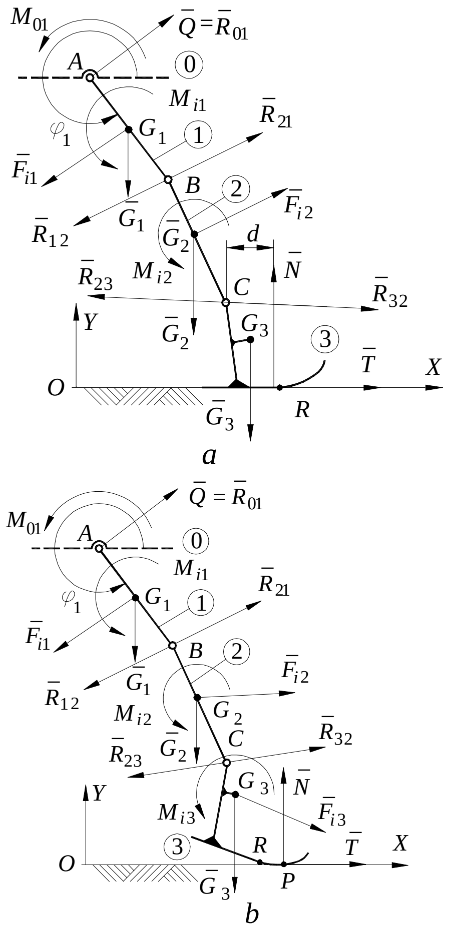

Fig. 7. Forces distribution in the leg mechanism.

$$\frac{d^2Y_{G2}}{dt^2}=\frac{d^2Y_A}{dt^2}+(x_{2G2}\cos\varphi_2-y_{2G2}\sin\varphi_2)\frac{d^2\varphi_2}{dt^2}$$

$$-(x_{2G2}\sin\varphi_2+y_{2G2}\cos\varphi_2)\left(\frac{d\varphi_2}{dt}\right)^2-$$

$$-AB(\cos\varphi_1\frac{d^2\varphi_1}{dt^2}-\sin\varphi_1\left(\frac{d\varphi_1}{dt}\right)^2);$$

$$\frac{d^2X_{G3}}{dt^2}=\frac{d^2X_A}{dt^2}-(x_{3G3}\sin\varphi_3+y_{3G3}\cos\varphi_3)\frac{d^2\varphi_3}{dt^2}$$

$$-(x_{3G3}\cos\varphi_3-y_{3G3}\sin\varphi_3)\left(\frac{d\varphi_3}{dt}\right)^2--AB(\sin\varphi_1\frac{d^2\varphi_1}{dt^2}$$

$$+\cos\varphi_1\left(\frac{d\varphi_1}{dt}\right)^2)-BC(\sin\varphi_2\frac{d^2\varphi_2}{dt^2}+\cos\varphi_2\left(\frac{d\varphi_2}{dt}\right)^2);$$

$$\frac{d^2Y_{G3}}{dt^2}=\frac{d^2Y_A}{dt^2}+(x_{3G3}\cos\varphi_3-y_{3G3}\sin\varphi_3)\frac{d^2\varphi_3}{dt^2}$$

$$-(x_{3G3}\sin\varphi_3+y_{3G3}\cos\varphi_3)\left(\frac{d\varphi_3}{dt}\right)^2+AB(\cos\varphi_1\frac{d^2\varphi_1}{dt^2}$$

$$-\sin\varphi_1\left(\frac{d\varphi_1}{dt}\right)^2)+BC(\cos\varphi_2\frac{d^2\varphi_2}{dt^2}-\sin\varphi_2\left(\frac{d\varphi_2}{dt}\right)^2).$$

The equations (4) are simultaneous solved with respect to the unknowns R_{01X}, R_{01Y}, R_{12X}, R_{12Y}, R_{32X} and R_{32Y}. The reaction force $\bar{R}_{ij}=R_{ijX}\bar{i}+R_{ijY}\bar{j}$ acts from the link (i) to the link (j).

Equations which expressing the equilibrium of the forces which act on the foot (3) are:

$$T-R_{32X}=0;\ N-R_{32Y}-m_3\,g=0; N\,d+T\,Y_C-m_3\,g\,(X_{G3}-X_C)=0. \quad (5)$$

These equations are solved with respect to the unknowns N, T and d. If $T>\mu N$, the foot slipped on the terrain. In this case, the input moment $M_{01}\neq 0$ must be applied to the thigh (1).

The magnitude of this moment is calculated by solving of the equations (4), where $R_{32X}<\mu N$. The sets of equations (4) and (5) are solved iteratively, until the difference between two successive iterations decreases under a certain limit.

2. The foot is supported with his front end on the terrain (Fig. 8.*b*).

The foot (3) may be in this position if a driving moment is applied in pair C, between links (2) and (3). The reaction forces from pair A and B are the solutions of the equations (6):

$$Q_X+F_{i1X}-R_{12X}+R_{01X}=0;\ Q_Y+F_{i1Y}-m_1\,g-R_{12Y}+R_{01Y}=0;$$

$$M_{01}+(F_{i1Y}-m_1\,g)(X_{G1}-X_B)-F_{i1X}\,(Y_{G1}-Y_B)+R_{01X}(Y_A-Y_B)+R_{01Y}(X_B-X_A)+M_{i1}=0;$$

$$F_{i2X}+R_{12X}+R_{32X}=0;\ F_{i2Y}-m_2\,g++R_{12Y}+R_{32Y}=0; \quad (6)$$

$$(F_{i2Y}-m_2\,g)(X_{G2}-X_B)-F_{12X}\,(Y_{G2}-Y_B)+R_{32Y}(X_C-X_B)-R_{32X}\,(Y_C-Y_B)+M_{i2}-M_{23}=0,$$

where $M_{01}=M_{23}=0$.

The unknowns of these equations are R_{01X}, R_{01Y}, R_{12X}, R_{12Y}, R_{32X} and R_{32Y}.

Equilibrium of the forces which act on the foot (3) is expressed by equations (7):

$$T-R_{32X}+F_{i3X}=0;\ N-R_{32Y}-m_3\,g+F_{i3Y}=0;$$

$$M_{23}+M_{i3}+N\,(X_P-X_C)+T\,Y_C+(F_{i3Y}-m_3\,g)\,(X_{G3}-X_C)-F_{i3X}(Y_{G3}-Y_C)=0. \quad (7)$$

Solutions of these equations are N, T and M_{23}. If $T>\mu N$, the foot slipped on the terrain and the robot overturns.

3.3. Optimum Design of the Foot

The bottom surface of the foot of a walking robot may have various shapes. These surfaces differ by the size of the flat surface and the forms of the front and rear cylindrical surfaces. The most adequate form of the bottom surface of the foot, i.e. the expressions of the directories of the front and rear cylindrical surfaces, is determined by optimization of some parameters. The objective [4] function, which is minimized in the optimization process may expressed:

- maximum angular velocity: $\frac{d\varphi_1}{dt}$, $\frac{d\varphi_2}{dt}$ or $\frac{d\varphi_3}{dt}$;
- maximum angular acceleration:

$$\frac{d^2\varphi_1}{d^2t}, \frac{d^2\varphi_2}{d^2t} \text{ or } \frac{d^2\varphi_3}{d^2t};$$

- the maximum driving forces or moments etc.

The design variables with respect to which the objective function is minimized are:

- the lengths *AB* and *BC* of the links (1) and (2),
- the coordinate x_{3R}, y_{3R} of the point *R*;
- the coefficients from the equations of the curve directories of the surfaces of the front and rear ends.

The minimization of the objective function is performed in the presence of constrains which expressed:

- the directory curves of the front and rear ends are tangent to the flat surface of the foot sole,
- the ordinates y_{3R} and y_{3T} of the points *R* and *T* respectively in which the directory curves are tangent to the flat surfaces are limit by the minimum flat surface of the foot.

4. EXAMPLE

The parametrical equations of the curve directories of the front end cylindrical surface are assumed as polynomial ones: $y_3 = \lambda$; $x_3 = \sum_{i=1}^{6} c_i \lambda^{i-1}$. Because the curve directories must pass through the point *R* defined by the coordinates x_{3R}, y_{3R}, results: $c_1 = x_{3R} - c_2 y_{3R} - c_3 y_{3R}^2 - c_4 y_{3R}^3 - c_5 y_{3R}^4 - c_6 y_{3R}^5$.

In the point *R*, the curve directories is tangent to the bottom flat surface of the sole:

$$\left.\frac{dy_3}{dx_3}\right|_{y=y_{3R}} = 0.$$

Whence it is results

$$c_2 = -2c_3 y_{3R} - 3c_4 y_{3R}^2 - 4c_5 y_{3R}^3 - 5c_6 y_{3R}^4.$$

The goal of the optimization problem is to calculate the coefficient of the polynomial directory and the dimensions of the leg mechanism links. The function with respect to which the leg design is optimized, i.e. the objective function, expressed the maximum value of the angular acceleration $\frac{d^2\varphi_1}{d^2t}$, $\frac{d^2\varphi_2}{d^2t}$ and $\frac{d^2\varphi_3}{d^2t}$, in terms of the coefficients of the polynomial directory and the dimensions of the leg mechanism.

The optimum shape of the front end of the foot is shown in Fig. 4, curve (1) [1, 8].

This polynomial curve has the following coefficients:

$$c_1 = 0.4140775,\ c_2 = 0.8095373,\ c_3 = -5.841186,$$

$$c_4 = 0,\ c_5 = 0.009742488,\ c_6 = 0.$$

The coordinate of the point *R* with respect to the mobile axes system x_3Cy_3 are:

$$x_{3R} = 0.4421264,\ y_{3R} = 0.06929674.$$

5. CONCLUSIONS

A conclusion section is not compulsory. Make sure that the whole text of your paper observes the textual arrangement on this page.

Low speed and high energy expenses characterize the walking robots. Usually, the increase of robot speed can be achieved by a very simple solution, namely increase of capability of the drives.

A rational solution is to use an analogy of the robot leg architecture with human leg. Of course, the complexity of the human leg is very large, so that many simplifications must be made. The use of an adequate surface shape of the rigid foot sole of the leg has the effect a betterment of the gait.

Because the leg is used for module of walking robot, the analysis and improvement of the proposal hardware architecture will be continued.

REFERENCES

[1] R. Fox, *Optimization Methods for Engineering Design*, Addison Wesley Publishing Comp, 1973.

[2] D., Goldberg *Genetic Algorithms in Search, Optimization and Machine Learning,* Addison Wesley Publishing Comp, 1999.

[3] I. Ion, I., Simionescu, A., Curaj, *Mobil Mechatronic System With Applications in Agriculture and Sylviculture,* The 2nd IFAC International Conference on Mechatronic Systems, pp. 321-327, 2002, December 8-12, Berkeley, USA.

[4] I. Simionescu, I. Ion., L. Ciupitu, *Mechanisms of Industrial Robots*, Tome I, Edit. AGIR, Bucharest, 2008.

[5] I. Simionescu, I. Ion, L. Ciupitu, *Mechanisms of Industrial Robots*, Tome II, Edit. AGIR, Bucharest, 2010.

[6] I. Ion, A. Marin, C. Doicin, C. Chirita, *Modeling and Simulation of Gait for MERO Modular Walking Robot crossing an unarranged Terrain*, Industrial Simulation Conference ISC2009, June 1–3, 2009, Loughborough University, United Kingdom, pp. 237–242, Proceedings of the ISC 2009.

[7] L. Vladareanu, I Ion, L. M. Velea, A. Curaj, D. Mitroi, T*he Hybrid Position And Force Control Of Robots With Compliance Function By Open Architecture Systems*, Proceedings of the 2009 International Conference On Information & Knowledge Engineering, Nevada, USA, The 2009 World Congress in Computer Science, Computer Engeneering, and Applied Computing, Published by CSREA Press, July 13–16, 2009, Las Vegas, Nevada, SUA, pp. 450–455.

[8] I. Ion, L. Vladareanu, A. Curaj, A. Vasile, *New Model of MERO Modular Walking Robots Support for Heavy Load Transportation*, Recent Advances in Electrical Engineering, A Series of Reference Books and Textbooks, Published by WSEAS Press, Proceedings of the 8th International Conference on Applications of Electrical Engineering (AEE '09), Houston, USA, pp. 141–146.

[9] M. Nishikawa et al, *European Patent Application* No. 0433091A2.

[10] S.M. Song, and J.K. Waldron, *An Analytical Approach for Gait Study and its Application on Wave Gait*, International Journal of Robotics Research, Vol. 6, No. 2, 1987, pp. 60–71.