MATHEMATICAL MODEL FOR DETERMINING THE ORIENTATION ERRORS GENERATED BY THE CONSTRUCTION OF THE PRISMATIC SUPPORTS

Nicoleta RACHIERU[1,*], Raluca Magdalena NIŢĂ[2]

[1)] PhD, Eng. Ec., Scientific Researcher II, Faculty Mechanics and Technology, University of Piteşti, Romania
[2)] PhD, Student, Eng., S.C. ICTCM S.A. – Mechanical Engineering and Research Institute, Bucharest, Romania

Abstract: *This paper presents aspects regarding the total errors which may occur in technological processing on machine tools, errors that affect the processing accuracy and also the quality of processed work-piece. In this context, technological devices have an important contribution with a category of errors, referred in the literature, installation errors that refer to work-piece orientation and fixing errors in devices. Apart from these, a special category of errors generated by the precision of execution of the support elements that are component part of the devices were identified and determined. For the analysis and calculation of this type of errors, for the short or long prismatic locators, a cinematic model was created, and also on its base a mathematic one.*

Key words: *technologic device, installation error, construction orientation error, prismatic locator, mathematical model.*

1. INTRODUCTION

In the actual context generated by the world economic crises and a serious of problems with which the humanity confront, starting with the natural resources exhaustion, till serious pollution problems, are impetuously necessary the development and improvement of new action directions for the development of all domains economic, social and technical. Modern manufacturing, marked by the accelerated generalization of the technical innovation, has determined the accentuation of product and request diversification regarding them performances and imposed demands related to the precision, reliability and productivity of the technologic process. The evolution of manufacturing systems, and of the technological devices, was marked by the necessity to ensure the appropriate level of precision and flexibility as bigger as possible in order to adapt quickly and effectively to changes to the production task.

In the manufacturing systems, the technologic devices (for processing, assembly and control) have an important place through the two basic functions: orientation and work-piece fixture. In a technological process, which normally takes place, the determining by statistical accuracy requires knowledge of the dispersion range and requires that all errors that appear during processing not to exceed tolerances. The analytical method allows the assessment of the errors by calculation and offers the possibility to reduce the total error by acting on the elements of technological systems.

[*] Corresponding author: Faculty Mechanics and Technology, University of Piteşti, Romania
Tel.: 0740676068
E-mail addresses: *nrachieru@yahoo.com* (N. Rachieru), *ralu_magda@yahoo.com* (R.M. Niţă)

2. TOTAL PROCESSING ERROR AND INSTALATION ERROR

In the literature a number of works exists, which establish relations for calculating the total error processing, error that influence the dimensional accuracy, shape and position deviations and surface quality of machined parts.

Thus in [6], for the processing on machine tools set in advance, when the dimensions are obtained automatically, the total error is expressed with the relationship:

$$\varepsilon_t = \varepsilon_i^R + \varepsilon_s^{\Sigma} , \tag{1}$$

where: ε_i^R is the total random error; ε_s^{Σ} represents the total systematic error.

The total random ε_i^R is calculated as a sum of four random errors with the relationship:

$$\varepsilon_{iR} = \sqrt{\varepsilon_i^2 + \varepsilon_s^2 + \varepsilon_m^2 + \varepsilon_{a\sigma}^2} , \tag{2}$$

where:

- ε_i, installation error, as a resultant of the orientation (ε_{or}) and fixture (ε_f) error; ε_s, tool dimensional error; ε_m, measuring error; $\varepsilon_{a\sigma}$, error because of the cutting deep variance and of the inhomogeneous material.

  In the systematic errors category are included:
- ε_g, error determined by the imprecision of the machine-tool;
- ε_d, error generated by the construction of the device component elements constructions, which generate systematic errors;
- ε_u, error determined by the tool wear;
- ε_{dt}, error determined by the tool thermal deformations (ε_{ts}), of the machine-tool (ε_{tmu}) and of the work-piece (ε_{tp});

- ε_e, error caused by the elastic deformations of the technologic system;
- ε_r, errors generated by the tool adjustment;

Total error, taking into consideration the two error categories is:

$$\varepsilon_t = \sqrt{\varepsilon_i^2 + \varepsilon_s^2 + \varepsilon_m^2 + \varepsilon_{a\sigma}^2} + \varepsilon_o + \\ + \varepsilon_d + \varepsilon_u + \varepsilon_{ts} + \varepsilon_{tm} + \varepsilon_{tp} + \varepsilon_e + \varepsilon_r \quad (3)$$

As in others works [1, 2, 7], the total error has the same dependence as in the mentioned errors and is expressed by:

$$\varepsilon_t = \sqrt{\varepsilon_i^2 + \varepsilon_v^2 + \varepsilon_r^2 + \varepsilon_c^2} + \varepsilon_{dt} + \varepsilon_u + \sum \varepsilon_{MU}\,, \quad (4)$$

where ε_t determines the dispersion interval (Δ) of the processed dimension ($\varepsilon_t \cong \Delta$).

The processing precision is provided if the following condition is respected:

$$\varepsilon_t \leq T, \quad (5)$$

where T is the imposed tolerance

The installation error (ε_i), has included in it the orientation (ε_o), fixture (ε_f) and the error related to the device construction error (ε_d). It is determined with the relationship:

$$\varepsilon_i = \sqrt{\varepsilon_o^2 + \varepsilon_f^2 + \varepsilon_d^2}\,, \quad (6)$$

For practical calculation in design, the value for the admissible installation error approximated with sufficient precision is: $\varepsilon_{iadm} = \left(\frac{1}{3} \ldots \frac{1}{2}\right) \cdot T$ and the real installation error is:

$$\varepsilon_i \leq \varepsilon_{iadm}\,. \quad (7)$$

Because of the factors that have an influence on the processing precision, it is obviously that the installing precision has the greatest influence and requires a complete study.

From the installation error component, the orientation and fixing ones represents one of the important factors in the design and analysis of the fixing systems.

Li and Melkote [8] have presented a model for improving the precision of orientation of the processed work-piece in a fixture system using a model with discrete contact.

Many others works have developed different methodologies for determining the fixing error influence, methods for positioning the work-piece on the locators and the possibilities to measure them [9].

In work [10], an algorithm is presented for determination of the processing errors for a prismatic work-piece, fixed on the base of the principle 3–2–1 and for a cylindrical work-piece using a V prism [11].

The work [12] has established and analysed the modality of error propagation from the disturbance in the locators, using a quadratic equation which gives a better approximation of the errors, in comparison with a linear equation. Positioning precision is a first request that must be taken into consideration at the design of the fixing systems and is related to the locators positioning precision and them position.

3. ORIENTATION ERROR GENRATED BY THE DEVICE CONSTRUCTION

At the use of technological devices, in general, but more frequently those for precision machining, it was found that although the error of work-piece orientation in the device, or $\varepsilon_{or} = 0$, the dimensions and conditions are not achieved within the acceptable limits due to errors generated by construction and operation of the locators elements, not considered in algorithm design.

In the papers [3, 4, 5] it is proposed the identification and calculation of construction orientation errors for a mobile bolt the bearing cell type (ε_{OCS}), one of the most commonly used in construction of the devices.

Are also presented the mobile long or short cylindrical bolt ( ) which are executed in different constructive variants and involves the same tapes of errors as the cylindrical mobile long bolt ( ), at which the specific error for the cone element is added.

In both cases, after the analysis of the cinematic model calculations relationships for the orientation error ε_{ocs} are established taking into consideration a seriess of parameters related to the construction and functioning of the locator.

Another type of locator used for exterior cylindrical surfaces processed or unprocessed is the long ( ), (Fig. 1a) or short prism ( ) (Fig. 1b), their analysis being presented in the paper.

In Fig. 2, the way of orientation and fixing of a work-piece with exterior cylindrical surface on a prismatic locator is represented.

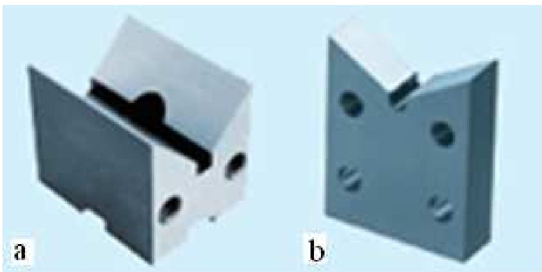

Fig. 1. Modular prisms:
a. long prism, *b*. short prisms.

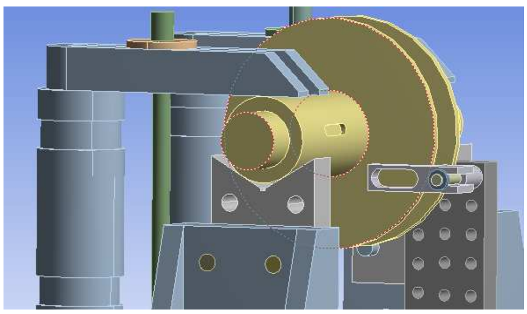

Fig. 2. The orientation and fixing for o cylindrical work-piece on a prism.

4. CINEMATIC AND MATHEMATIC MODEL FOR THE ORIENTATION ERROR CALCULATION FOR THE CONSTRACTION OF THE PRISMATIC LOCATOR

In the short or long prisms, it is found that the orientation errors depend not only on the locator element and are function of the constructive errors of the prism and, at a certain moment, they may be added.

For establishing the cinematic and mathematic models the following stages are covered:

- Stage I – identification of the prism constructions errors;
- Stage II – calculation of the orientation-positioning errors for every constructive deviation on each of the principal directions *x*, *y*, *z*;
- Stage III – vectorial adding of the results obtained in the Stage II.

For the first stage, in the case of the short or long prismatic locator, four types of constructive deviations were indentified:

a) Deviation of symmetry.

b) Deviation from the half-angle of the prism.

c) Deviation from the parallelism with the z axes.

d) Deviation from the parallelism with the bases plane.

In the second stage, are obtaining for every particular case, specific relations, which give values for the displacements on every axis.

a) Only displacement on axes *x* and *y* are obtained; In the right triangle *CC'P* (Fig.3),

$$\tan\frac{\alpha}{2} = \frac{CP}{C'P} = \frac{x/2}{C'P} = \frac{x}{2C'P}, \tag{8}$$

From were results:

$$C'P = \frac{x}{2}\cdot\cot\frac{\alpha}{2}, \tag{9}$$

From the triangles CPC' and OQO' equality the following relationships are obtained:

$$\begin{cases} \Delta x = \dfrac{x}{2} \\ \Delta y = \dfrac{x}{2}\cdot\cot\dfrac{\alpha}{2} \end{cases}, \tag{10}$$

where: *x* – symmetry deviation; α – the prism open angle; Δ*x* – displacement along x axes; Δ*y* – displacement along *y* axes;

b) Also in this case displacements are obtained only on the axes *x* and *y* (Fig. 4);

In the rectangular Δ*CBO* and Δ*CB'O'*,

$$OC = R\cdot\frac{1}{\sin\frac{\alpha}{2}}$$

and

$$O'C = R\cdot\frac{1}{\sin\frac{\alpha+\Delta\alpha}{2}}, \tag{11}$$

$$CB = R\cdot\cot\frac{\alpha}{2},$$

and

$$CB' = R\cdot\cot\frac{\alpha+\Delta\alpha}{2}, \tag{12}$$

$$\begin{aligned} BB' = OO' &= R\cdot\left(\cot\frac{\alpha}{2} - \cot\frac{\alpha+\Delta\alpha}{2}\right) = \\ &= R\cdot\frac{\sin\frac{\Delta\alpha}{2}}{\sin\frac{\alpha}{2}\cdot\sin\frac{\alpha+\Delta\alpha}{2}}. \end{aligned} \tag{13}$$

In the triangle *OO'C* the two formulas are applied known for the area calculation. Thus:

$$A_{CO'O} = \frac{O'C\cdot OC\cdot\sin\frac{\Delta\alpha}{2}}{2} = \frac{OC\cdot O'Q}{2}. \tag{14}$$

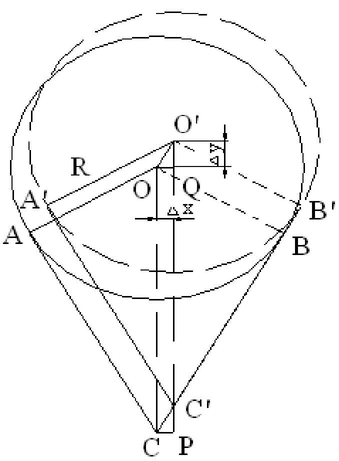

Fig. 3. Mathematic model for the prism deviation symmetry.

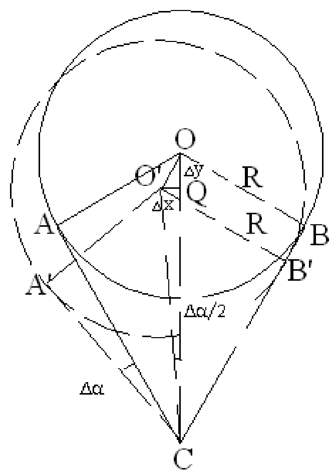

Fig. 4. Cinematic model for the deviation from the prism half-angle.

From the relation (14), we obtain:

$$O'Q = O'C \cdot \sin\frac{\Delta\alpha}{2} = R \cdot \frac{\sin\frac{\Delta\alpha}{2}}{\sin\frac{\alpha+\Delta\alpha}{2}}. \qquad (15)$$

In the right triangle OO’Q,

$$OQ = \sqrt{O'O^2 - O'Q^2}\ .$$

Using the relationships (13) and (15), it results:

$$OQ = R \cdot \frac{\sin\frac{\Delta\alpha}{2}}{\sin\frac{\alpha+\Delta\alpha}{2}} \cdot \cot\frac{\alpha}{2}. \qquad (16)$$

The relations for the displacement on the two axes are:

$$\begin{cases} \Delta x = R \cdot \dfrac{\sin\frac{\Delta\alpha}{2}}{\sin\frac{\alpha+\Delta\alpha}{2}} \\ \Delta y = R \cdot \dfrac{\sin\frac{\Delta\alpha}{2}}{\sin\frac{\alpha+\Delta\alpha}{2}} \cdot ctg\dfrac{\alpha}{2} \end{cases}, \qquad (17)$$

where:
R is the radius of the cylindrical surface, tangent to the prism openness;
α – angle of the prism openness;
$\Delta\alpha$ – deviation of the prism angle;
Δx – displacement on the x;
Δy – displacement on the y;

c) Deviation from the parallelism with the *z* axes, which involves a rotation in the three-dimensional space along *OY* axe, which means a displacement on the directions *x*, *y*, *z*.

It can be observed that after the rotation of the two half of the prism, the cylindrical surface axes laid tangent to the openness of the (prism axes) will remain parallel to the intersection line of the two lateral surface of the prism (Fig.5).

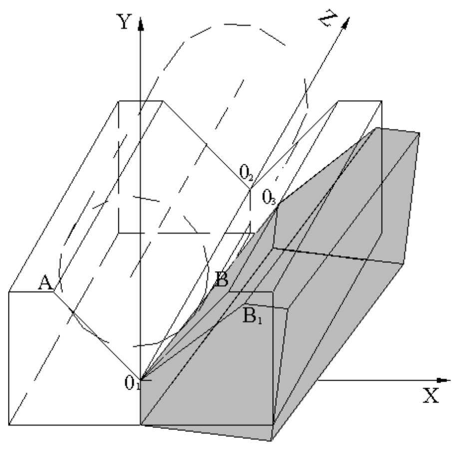

Fig. 5. Cinematic model for the deviation from the parallelism with *z* axes.

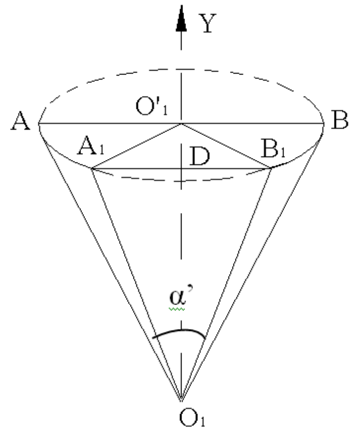

Fig. 6. The rotation cone of the point from the pyramid edge.

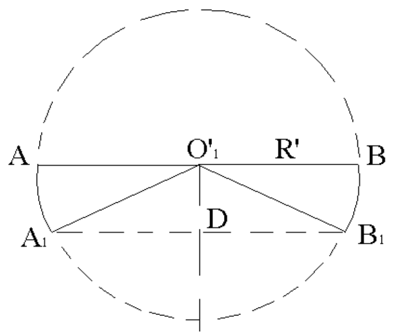

Fig. 7. The base of the rotational cone.

Points *A* and *B* will displace a circle arch equal to the rotation angle and will be materialised in the space with rotation cone (Fig. 6).

In the rectangular triangle O'_1B_1D (Fig. 7),

$$\sin(90 - \frac{\Delta\beta}{2}) = \frac{DB_1}{O_1^{'}B_1} = \frac{DB_1}{R^{'}} \qquad (18)$$

where:

$$DB_1 = R^{'} \cdot \sin(90 - \frac{\Delta\beta}{2})$$

Thus, in $\Delta O'_1BO_1$ (Fig. 6),

$$\sin\frac{\alpha}{2} = \frac{O_1^{'}B}{O_1B} = \frac{R^{'}}{O_1B},$$

where:

$$O_1B = \frac{R^{'}}{\sin\frac{\alpha}{2}} = O_1B_1, \qquad (19)$$

In ΔDB_1O_1,

$$\sin\frac{\alpha}{2} = \frac{DB_1}{O_1B_1} = \frac{R'\sin(90 - \frac{\Delta\beta}{2})}{R'/\sin\frac{\alpha}{2}} = \\ = \sin\frac{\alpha}{2} \cdot \sin(90 - \frac{\Delta\beta}{2}). \qquad (20)$$

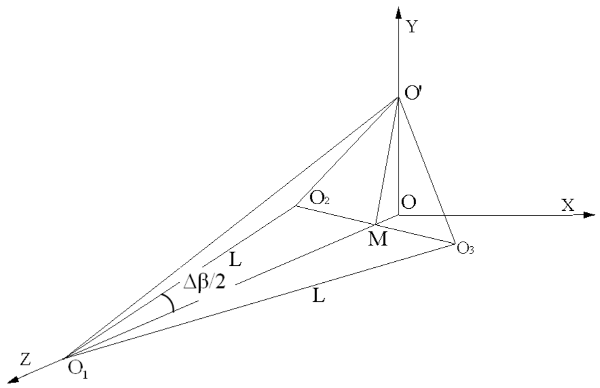

Fig. 8. The pyramid of the two planes of the prism.

In Fig. 8 a pyramid is represented formed by the two lateral surfaces of the prism, the intersection line and *ZOX* plane, rotated with 180^0 for the easiness of the demonstration.

Also for simplifying the calculation the origin of the axis system will be translated in the point that materialised the projection of *O'* and *O* respectively on the *ZOY* plane.

With these elements, $\Delta O_1O_2O_3$ and $\Delta O'O_2O_3$ are isosceles.

In $\Delta\, O_1O_2M$,

$$O_2M = O_1O_2 \cdot \sin\frac{\Delta\beta}{2} = L \cdot \sin\frac{\Delta\beta}{2}$$

and

$$O_1M = L \cdot \cos\frac{\Delta\beta}{2}.$$

The angle $O_2O'O_3$ is equal to the angle $A_1O_1B_1$ and measure α'. So, in $\Delta O_2MO'$,

$$\sin\frac{\alpha^{'}}{2} = \frac{MO_2}{O^{'}O_2}.$$

It results:

$$O'O_2 \frac{L\sin\frac{\Delta\beta}{2}}{\sin\frac{\alpha'}{2}} = \frac{L\sin\frac{\Delta\beta}{2}}{\sin\frac{\alpha}{2}\cdot\sin(90-\frac{\Delta\beta}{2})}. \qquad (21)$$

From the right triangle $\Delta O_1O_2O'$, the following relations are obtained:

$$\left(O_1O^{'}\right)^2 = L^2 + L^2 \cdot x \cdot C^2,$$

where:

$$C = \frac{\sin\frac{\Delta\beta}{2}}{\sin\frac{\alpha}{2}\cdot\sin(90-\frac{\Delta\beta}{2})}.$$

Thus, in the right triangle $\Delta MO_2O'$:

$$(O^{'}M)^2 = (O^{'}O_2)^2 - (MO_2)^2 = L^2 \cdot C^2 - L^2 \cdot \sin^2\frac{\Delta\beta}{2}. \qquad (22)$$

Applying the cosine theorem in $\Delta O_1MO'$,

$$(O_1O^{'})^2 = (O_1M)^2 + (O^{'}M)^2 - 2\cdot O_1M \cdot O^{'}M \cdot \cos\gamma$$

where

$$\gamma = \sphericalangle O_1MO, \qquad (23)$$

$$\cos\gamma = \frac{L^2 + L^2\cdot C^2 - L^2\cdot\cos^2\frac{\Delta\beta}{2} - L^2\cdot C^2 + L^2\cdot\sin^2\frac{\Delta\beta}{2}}{-2L\cos\frac{\Delta\beta}{2}\sqrt{L^2\cdot C^2 - L^2\sin^2\frac{\Delta\beta}{2}}}. \qquad (24)$$

By reduction, we obtain the relationship:

$$\cos\gamma = \frac{L^2 - L^2\cdot\cos^2\frac{\Delta\beta}{2} + L^2\cdot\sin\frac{\Delta\beta}{2}}{-2L\cos\frac{\Delta\beta}{2}\sqrt{L^2\cdot C^2 - L^2\sin\frac{\Delta\beta}{2}}}. \qquad (25)$$

The angle $O^{'}MO = \gamma^{'}$. With this notation we have:

$$\gamma^{'} = 180^\circ - \gamma \text{ and } \cos\gamma^{'} = -\cos\gamma = OO^{'}/O^{'}M$$

It results:

$$OO^{'} = O^{'}M\cdot\cos\gamma =$$

$$= \sqrt{L^2\cdot C^2 - L^2\cdot\sin^2\frac{\Delta\beta}{2}} \cdot \frac{L^2 - L^2\cdot\cos^2\frac{\Delta\beta}{2} + L^2\cdot\sin^2\frac{\Delta\beta}{2}}{2L\cos\frac{\Delta\beta}{2}\sqrt{L^2\cdot C^2 - L^2\sin^2\frac{\Delta\beta}{2}}}.$$

By simplification it is obtained:

$$OO^{'} = \frac{L - L\cos^2\frac{\Delta\beta}{2} + L\sin^2\frac{\Delta\beta}{2}}{2\cos\frac{\Delta\beta}{2}} = L\frac{\sin^2\frac{\Delta\beta}{2}}{\cos\frac{\Delta\beta}{2}}, \qquad (26)$$

From the rectangular $\Delta MOO'$, it is calculated

$$MO^2 = (O^{'}M)^2 - (OO^{'})^2$$

and we have:

$$MO^2 = L^2\cdot C^2 - L^2\cdot\sin^2\cdot\frac{\Delta\beta}{2} - L^2\cdot\left(\frac{\sin^2\frac{\Delta\beta}{2}}{\cos\frac{\Delta\beta}{2}}\right)^2 = L^2\cdot C^{'2}. \qquad (27)$$

From Figs. 8 and 9, displacements on the three axis are:

$$\begin{cases} \Delta x = MO_2 = L\sin\frac{\Delta\beta}{2} \\ \Delta y = MO = L\cdot C \\ \Delta z = OO' = L\frac{\sin^2\frac{\Delta\beta}{2}}{\cos\frac{\Delta\beta}{2}} \end{cases}. \qquad (28)$$

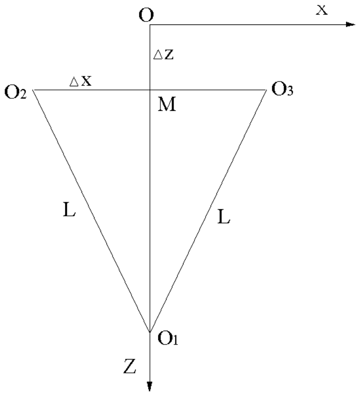

Fig. 9. The pyramid base representation.

In the relations (28), L – is the contact length between the two half prisms and $\Delta\beta$ is the angular displacement obtain through the rotation along *OY* axes.

d) Deviation from parallelism from the settle position involves o rotation in the three dimensional space around the *Ox* axes and also a displacement on the principal directions *x*, *y*, *z*, calculated similarly as in point *C*.

The deviations presented in the four cases can took place simultaneously or separately in function of execution precision of the two half-prisms.

Because of these, the displacements created by the three axes must be added.

Because these displacement can take place in both directions on every axes for o correct appreciation will be used a vectorial sum and not an algebraic one, which will offer the possibility to fulfil the proposed purpose: to take in consideration these errors and to eliminate them as much as possible.

It is obtained: $\vec{\Delta} = \overrightarrow{\Delta x} + \overrightarrow{\Delta y} + \overrightarrow{\Delta z}$ where, for $i = 1 - n$ identified cases:

$$\begin{cases} \overrightarrow{\Delta x} = \sum_{i=1}^{n} \overrightarrow{\Delta x_i} \\ \overrightarrow{\Delta y} = \sum_{i=1}^{n} \overrightarrow{\Delta y_i} \\ \overrightarrow{\Delta z} = \sum_{i=1}^{n} \overrightarrow{\Delta z_i} \end{cases} . \quad (29)$$

5. CONCLUSIONS

Taking into consideration that the device is considered a subsystem of the technological system, constituting a unit from technologic, constructive and functional point of view, which establishes and maintains the orientation of the half finished product or of the tools with possibilities to assume functions of the machine-tool or operator, it is necessary a great attention starting from the design phase regarding the necessary precision and the quality of the component elements. The calculations of the orientations errors generated by the construction of the devices locators (ε_{ocs}), because of the dimensional deviation, form and positions of the component elements, makes it possible the more objective appreciation of the orientation errors and also of the installation one.

Failure to take into account the errors generated by the locators element construction may compromise the device realization and especially the precision.

The mathematic model proposed in the paper presents a high level of complexity and was created for a general case. Its utilization for every particular case involves important simplifications. Thus, starting from this mathematic model and using the great advantages which are assumed by the computer using, it can be created specialised software which will increase the precision and the rapidity of the determination and also to permit a simulation of the studied phenomena.

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