Proceedings in Manufacturing Systems, Volume 6, Issue 4, 2011 ISSN 2067-9238

THE INFLUENCE OF TOTAL ELASTIC DISPLACEMENT OF TRANSLATION JOINTS ON GANTRY ROBOT ACCURACY

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Abstract: *High speeds and accuracy of robotic manipulators are current requirements to obtain improved performances of industrial robots. However, higher speeds and accuracy are greatly influenced by elastic behavior of mechanical structure of industrial robots. For this purpose an original approach on gantry robots volumetric accuracy evaluation and P-joints elastic behavior specific influence on IR's volumetric accuracy quantifying method have been developed. Following up these issues the paper successively presents in two parts: the original mathematic modeling for gantry IR's overall elastic behavior and specific algorithm / quantifying method of P joints elastic displacements influence on IR's volumetric accuracy (both of them generally valid for any kind of gantry robot and P-joint specific internal design), specific performances of an usual model of Güdel gantry robot type and respectively it's virtual prototype already developed by authors for performing specific applied calculus related to this robot's P joints / overall elastic behavior modeling, the mathematic model of overall gravitational and inertial load distribution, specific mathematic algorithm for loading behavior evaluation related to robot's P-joints partially assemblies and each included cam following guiding components.*

Key words: *industrial robots, accuracy, volumetric error, elastic displacements, analytical model, FEM.*

1. INTRODUCTION

In this paper the authors propose an analytical model for gantry robot's volumetric accuracy evaluation based on elastic displacements of joints. To properly define the virtual model for robot's overall FEM analysis was necessary to determine first the stiffness of the robot's translational joints including cam-followers components. For this purpose using the mathematical model, developed by Nicolescu et al. (2010), the overall loading of the Gantry robot has been reduced to corresponding axial and radial direction for each cam-follower, the specific loads on each cam-follower being determined in accordance with specific design for each translational joint. To express the displacements in radial and axial direction of ball bearings, the authors have used the mathematical background presented by De Tedric et al, (2007) in which the elastic displacements are expressed as δa (ball bearing axial displacement) and δr (ball bearing radial displacement). By using FEM analysis, the elastic behavior of the gantry robot structure subjected to static load was analyzed and errors induced by structural elements elastic displacements were revealed. By studying results obtained by using numerical applications developed by authors, several possible design alternatives, the optimum design solution having the highest stiffness for robot's structural elements can be identified.

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2. 1. Calculation model for determining the end-effecter's positioning error matrix

Positioning errors are the sum effects of elastic movements of joints, structural elements and kinematic chains, but the biggest share is considered to have the elastic displacements of joints guiding elements**.** Thus, further, it will be presented the full mathematical model corresponding to the Gudel Fp4 gantry robot.

According to Denavit-Hartenberg algorithm, the total transformation matrix for Gantry robot positioning system (Figs.1 and 2) is:

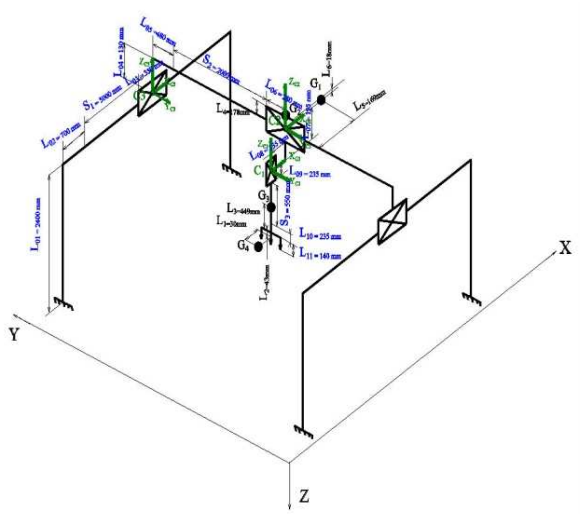
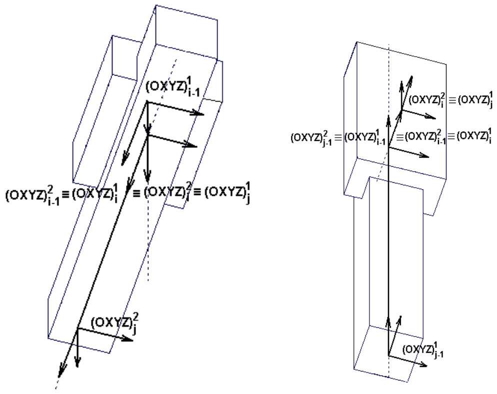


Fig. 1. Kinematics scheme for a Gantry robot.



a b

Fig. 2. Translation joint general model with fixed frame: *a* – for *Z* axis and mobile reference system; *b* – for *X* and *Y* axis.

The mathematical model, according to Denavit-Hartenberg algorithm, can be written as follows:

$$T_{(4,0)}^{HD} = T_{(1,0)}^{HD} \cdot T_{(2,1)}^{HD} \cdot T_{(3,2)}^{HD} \cdot T_{(4,3)}^{HD} = \begin{pmatrix} 0 & 0 & 0 & L_{02}+S_1-L_{08} \\ 0 & -1 & 0 & L_{05}+S_2+L_{06} \\ 0 & 0 & -1 & L_{01}+L_{04}-L_{07} \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & S_3+L_{10}+L_{09}+L_{11} \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 & 0 & L_{02}-L_{08}+S_1 \\ 0 & -1 & 0 & L_{05}+L_{06}+S_2 \\ 0 & 0 & -1 & L_{01}-L_{10}+L_{02}-L_{07}-L_{09}-S_3-L_{11} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

The matrix expression written above is valid for an ideal robot, but to have a form that respects the reality, it is necessary to take into account the errors that exist as a result of elastic displacements.

Taking into account the effect of errors for each prismatic joint, we can write:

$$T_{P-\text{joint}} = E_i^D E_i^C T_i^{HD} E_j^D . \tag{2}$$

where E_i^D and E_j^D factors represent the errors given by elastic displacements in structural components and geometric errors, so knowing that errors given by elastic displacements in guiding systems have the biggest influence (70 %), the last two factors named will be omitted in following calculation, so according to what have been just said, the mathematical model corresponding to a real robot is:

$$T_{gantryIR}^{real} = T_I T_1^{HD} T_{II} E_1^D T_{III} T_{IV} T_2^{HD} \cdot$$
$$\cdot T_V E_2^D T_{VI} E_3^D T_{VII} T_3^{HD} . \tag{3}$$

Where $T_{\mathrm{I}} \ldots T_{\mathrm{VII}}$ are depending to each type of robot architecture, for Gantry robot presentenced in this paper being as it follows

$$T_{\mathrm{I}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_{01} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

$$\mathrm{T_{II}} = \begin{pmatrix} 1 & 0 & 0 & L_{06} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

$$\mathrm{T_{III}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

$$\mathrm{T_{IV}} = \begin{pmatrix} 1 & 0 & 0 & L_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

$$\mathrm{T_{V}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

$$\mathrm{T_{VI}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_{07} \\ 0 & 0 & 1 & -L_{09}-S_3+L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

$$\mathrm{T_{VII}} = \begin{pmatrix} 1 & 0 & 0 & L_{08} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

And E_i^D matrix is:

$$E_i^D = R_{xi} \cdot R_{yi} \cdot R_{zi} \cdot T_{\varepsilon i} . \tag{10}$$

to consider particular effects introduced by roll, pitch, yaw angular elastic displacements and respectively *x*, *y*, *z* linear elastic displacements of every P-joint, three (4×4) rotational matrices and a single (4×4) cumulative translation matrix of errors has been included in the mathematical model to permit homogenous transformation between the coordinate system attached to "ideal" (undeflected) and respectively "real" (deflected) P-joint (Fig. 3 and Table 1). Where the mathematical expression factors are as it can be seen bellow [1, 3, and 6]:

$$R_{xi} = \begin{pmatrix} \cos(\alpha_{ix}) & -\sin(\alpha_{ix}) & 0 & 0 \\ \sin(\alpha_{ix}) & \cos(\alpha_{ix}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

$$R_{yi} = \begin{pmatrix} \cos(\alpha_{iy}) & 0 & \sin(\alpha_{iy}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\alpha_{iy}) & 0 & \cos(\alpha_{iy}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{12}$$

$$R_{zi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_{iz}) & -\sin(\alpha_{iz}) & 0 \\ 0 & \sin(\alpha_{iz}) & \cos(\alpha_{iz}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{13}$$

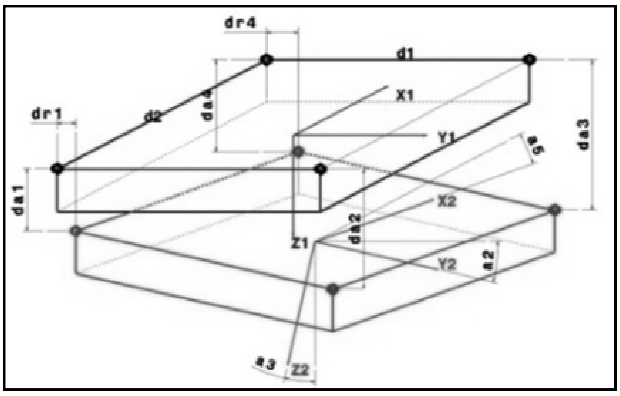


Fig. 3. General deformed/un-deformed model for a P-joint.

Table 1

Matrix components

i	α_{xi}	α_{yi}	α_{zi}	t_{xi}	t_{yi}	t_{zi}
1	α_{31}	α_{21}	α_{51}	α_{61}	α_{41}	α_{11}
2	α_{52}	α_{32}	α_{22}	α_{12}	α_{62}	$\alpha_{42}+L_{02}+S_1+L_{03}$
3	α_{53}	α_{23}	α_{33}	α_{13}	$\alpha_{43}-L_{05}+S_2+L_{06}$	α_{63}

$$T_{\varepsilon i} = \begin{pmatrix} 1 & 0 & 0 & t_{xi} \\ 0 & 1 & 0 & t_{yi} \\ 0 & 0 & 1 & t_{zi} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (14)$$

Elastic deformations occur preferentially in bearings, thus determining the elastic deformation of the joints is necessary to know the elastic displacements of the ball bearings on the radial and axial directions. Bearings elastic deformations can be determined by using specific developed numerical models (Chapter 6) [1, 5, and 6].

3. MAIN FEATURES OF STUDIED ROBOT

Gantry robots, the FP range offered by Güdel (Fig. 4), can successfully be integrated on any robotic application, being able to integrate in applications as machine building, industry applications, pharmaceutical or food industry. Also performs positioning accuracy and great speed despite the large volume of work Motor drive is only electric, moving the axles being carried by a pinion-rack type gear. Modular design allows the robot to adapt to any application, assembling it being made quickly and easy. The modules have a special construction that allows obtaining a high stiffness at the same time avoiding getting heavy and oversized structures. The need to communicate and interact with the application systems, near the robot, is a very important aspect, so the interest is that the robot, which will be selected for integration into the application, to have a controller that can easily interface with peripheral systems, but which, at the same time, offers the possibility to be easily programmed, the user interface is necessary to be more friendly to be used by a less experienced programmer. Controller that Güdel uses for its FP range robots is KUKA KM C2G; it meets all the requirements listed above [5].

Virtual prototype derived from Fig. 4 was created as part of a research contract and discussions of second part of the paper will use data obtained from studying the behavior of the prototype structure to static loads.

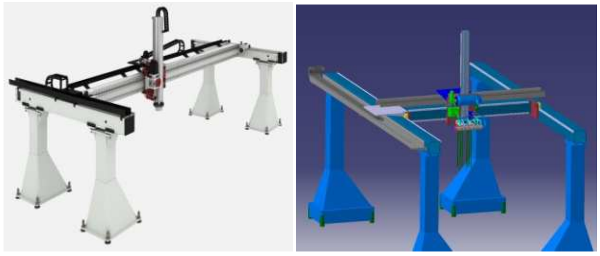


Fig. 4. Fp4 Gudel gantry robot: Gudel gantry robot Virtual Prototype [2, 4, and 5].

In the following rows are given the technical characteristics of the robot described above:

- Payload: 1000 [N];
- Stroke on X axis : 5 000 [mm];
- Stroke on Y axis: 2 000 [mm];
- Stroke on Z axis: 1 500 [mm];
- X and Y axis speed: 1 12.5 [m/min];
- Z axis speed: 67.5 [m/min];
- X axis acceleration: 1.5 [m/min²];
- Y axis acceleration: 4 [m/min²];
- Z axis acceleration: 2.5 [m/min²];
- Repeatability: ≤ 0.05 [mm] [2].

3.1. Construction of Güdel Fp4 robot P-joints

In Fig. 5 the technical solutions adopted by Güdel for construction of the P-joints used on its robots are presented:

In the second paper part is intended to determine the loads on each element of the guidance; this is the first step in finding the elastic displacements of the guiding elements. To guide translation couplings, Güdel, using rollers, how to mount them, depends mainly by the robot payload, the higher it is, the rollers mounting scheme should provide an increased stiffness. In Fig. 5, the rollers mounting solution, is quite simple, that is because of the relative small payload of the studied robot. So knowing the mounting solution for the track rollers, can proceed to calculate the forces on each roller [5].

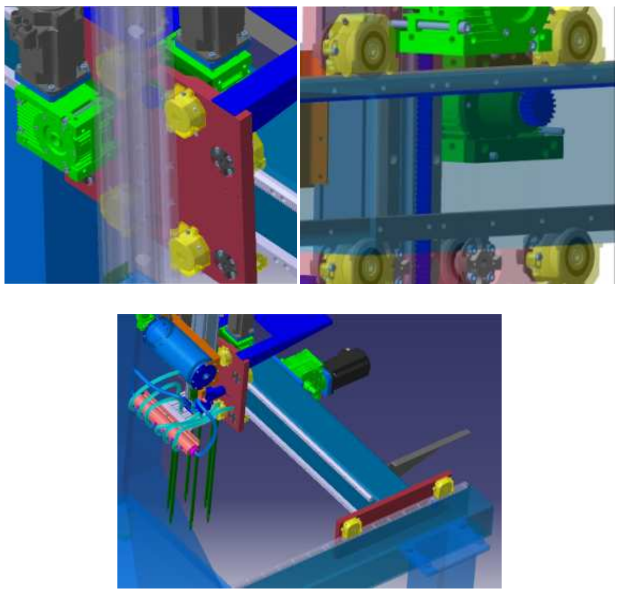
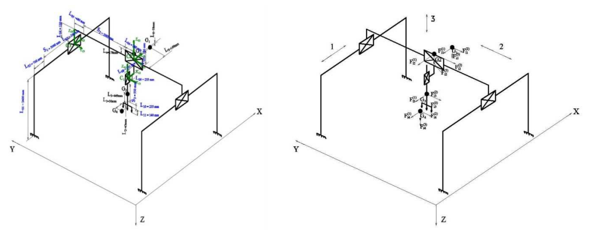


Fig. 5. Virtual prototype for Güdel X, Y and Z P-joints.



a b

Fig. 6. *a* – Kinematic scheme for studied gantry type IR; *b* – centers of gravity position for each P-joints.

4. OVERALL SYSTEM LOADING EVALUATION

Forces acting on the guiding elements are the result of inertial effects, unbalanced gravity centers and force of gravity. Kinematics scheme in Fig. 6a is consistent with the actual dimensions of the robot and respects the conventions of Hartenberg-Denavit algorithm, moreover, one other important aspect for determining loads on joints, is to calculate the position of center of gravity of the P-joints (Fig. 6b), for their determination were used CATIA V5 functions [7].

4.1. Equations to determine loads on the guidance system for *Z* axis

To determine the loading value is first necessary to determine the inertial forces. Equations and values of the centers of gravity, inertial forces G_i where determined using the mathematical model presented in former paper of the authors [6].

Thus, loads are determined in the kinematic couplings, we can proceed to determine the forces on each guidance element and then evaluating the elastic displacements and their influence on gantry robot volumetric accuracy [1].

In Fig. 7 are given resulting loads on each cam follower in hand. It is clear that loading resulting in *Z* axis will be supported by corresponding actuator of this axis, since the displacement is along the *Z* axis. Loads results in the Y axis are taken over by the roller on the radial direction and the resulting loads along the *X* axis are taken in the direction of roller axial.

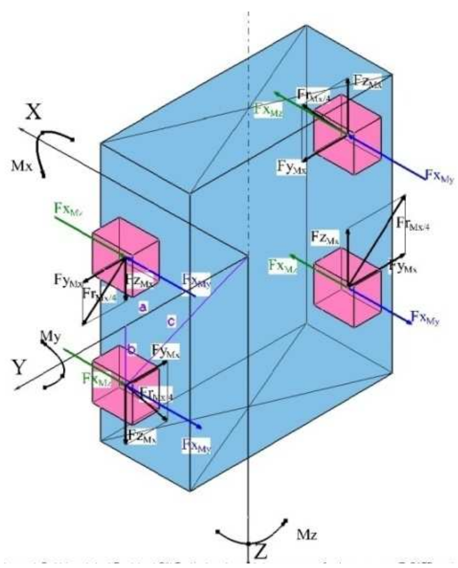


Fig. 7. Distribution of forces for *Z* P-joint.

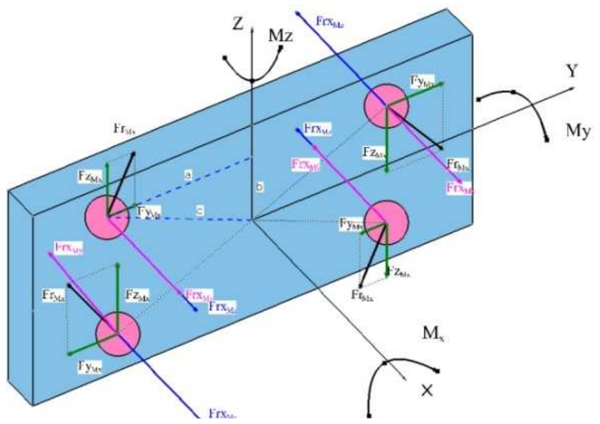


Fig. 8. Distribution of forces for Y P-joint.

In Fig. 8 the distribution of forces on each axis of coordinate corresponding to robot *Y* axis is observed. Forces generated by the moments *Mz* and *My* result on the direction of axes of reference, making it different when force generated by *Mz*, it is distributed on the guidance rollers radial direction that corresponds to the assembly moving direction.

$$F_{rMx3} := \frac{\sum M_{x3}}{4c}. \tag{15}$$

$$Fy_{Mx3} := Fr_{Mx3} \cdot \cos(\theta). \tag{16}$$

$$Fx_{Mz3} := \frac{\sum M_{z3}}{4b}. \tag{17}$$

$$Fx_{My3} := \frac{\sum M_{y3}}{4a}. \tag{18}$$

$$F_{rad3} := Fy_{Mx3}, \tag{19}$$

$$F_{ax3} := Fx_{Mz3} + Fx_{My3}. \tag{20}$$

$$\mathrm{Fr_{Mx}} := \frac{\sum \mathrm{M_{x2}}}{2\mathrm{c}}. \tag{21}$$

$$\mathrm{Fz_{Mx2}} := \mathrm{Fr_{Mx}} \cdot \cos(\varphi). \tag{22}$$

$$\mathrm{Fr_{My2}} := \frac{\sum \mathrm{M_{y2}}}{4\mathrm{b}}. \tag{23}$$

$$\mathrm{Fr_{Mz2}} := \frac{\sum \mathrm{M_{z2}}}{4\mathrm{a}} \quad .(24)$$

$$\mathrm{F_{rad2}} := \mathrm{Fz_{Mx2}} + \frac{\sum \mathrm{F_{z2}}}{2}. \tag{25}$$

Additional to forces generated by the overturning moments, on the radial direction of the guiding rollers, came, the forces of gravity acting proper axis *Y*.

$$F_{ax2} := Fr_{My2} + Fr_{Mz2}. \tag{26}$$

Similar to *Z* and *Y* axes are used for construction of the X axis (Fig. 9), using the same, also maximum dynamic carrying capacity of maximum remains unchanged. Note in Fig. 5 the distribution of the force, generated by the moment *My* along the coordinate axes and the loads generated by the moments *Mx* and *My* respectively resulting in axial direction of guide rollers [1].

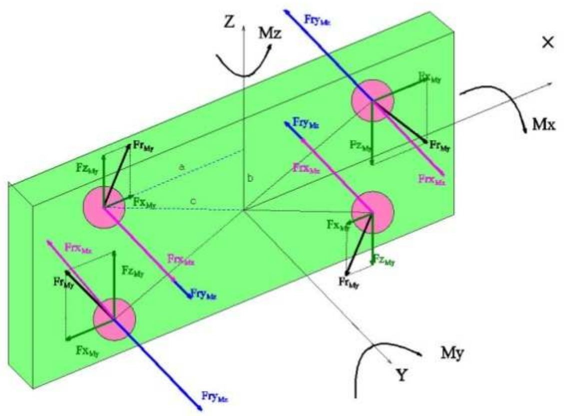


Fig. 9. Distribution of forces for X P-joint.

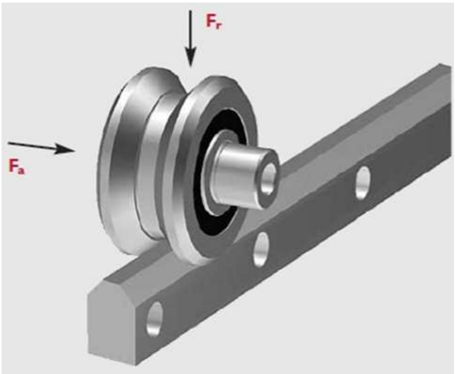


Fig. 10. Loads distribution o on an cam-follower.

$$Fr_{My1} := \frac{\sum M_{y1}}{3c}, \tag{27}$$

$$Fz_{My1} := Fr_{My1} \cdot \sin(\beta), \tag{28}$$

$$Fry_{Mz1} := \frac{\sum M_{z1}}{4a}, \tag{29}$$

$$Frx_{Mx1} := \frac{\sum M_{x1}}{4b}, \tag{30}$$

$$F_{rad1} := Fz_{My1} + \frac{\sum F_{z1}}{4}, \tag{31}$$

$$F_{ax} := Fry_{Mz1} + Fr_{Mx1}. \tag{32}$$

In the first part of the work loads for each translation joint were determined, next the elastic displacements on each cam follower will be determined. The loads on guidance elements are being defined as an axial load plus a radial load as shown in the following Fig. 10.

The cam follower elastic deflection are basically the radial and axial deflections from the bearing inside of it, so, the forces on each guide roller being known and the mathematical model for the particular gantry robot, taken as example in this paper, being as well known, it remains only to edify the radial and axial deflection on bearings as it can be seen in the section 6.

5. ANALITICAL MODEL FOR ROBOT P-JOINTS TOTAL DEFLECTION

The influence of IR's joints and links elastic behavior on IR's volumetric accuracy can be expressed by a total error matrix [ε]:

$$[\varepsilon] = T_{IR}^{real} - T_{(4,0)}^{HD}. \tag{32}$$

The Δ_i^J term can be expressed as follows:

$$[\Delta_i^J] = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{33}$$

where

$$n_x = \cos a_5 \cos a_2, \tag{34}$$

$$n_y = \sin(a_3)\sin(a_5)\cos(a_2) + \cos(a_3)\sin(a_2), \tag{35}$$

$$N_z = -\cos(a_3)\sin(a_5)\cos(a_2) + \sin(a_3)\sin(a_2), \tag{36}$$

$$o_x = \sin(a_5), \tag{37}$$

$$o_y = -\sin(a_3)\cos(a_5), \tag{38}$$

$$o_z = \cos(a_3)\cos(a_5), \tag{39}$$

$$a_x = \cos(a_5)\sin(a_2), \tag{40}$$

$$a_y = \sin(a_3)\sin(a_5)\sin(a_2) - \cos(a_3)\cos(a_2), \tag{41}$$

$$a_z = -\cos(a_3)\sin(a_5)\sin(a_2) - \sin(a_3)\cos(a_2) \tag{42}$$

$$p_x = 0, \tag{43}$$

$$p_y = a_1, \tag{44}$$

$$p_z = a_4. \tag{45}$$

Knowing the loads corresponding to radial and axial direction for each cam-follower and internal load distribution inside each cam-follower, the resulting linear and angular displacements of each robot's mobile element representing the general deformed/un-deformed model for a P-joint, assumed to be vertical (a_1), horizontal (a_4), pitching angle (a_2), rolling angle (a_3) and yawing angle (a_5), may be expressed as Fig. 3 shows too by equations (46)–(53) [3]:

$$A_1 = \frac{d_{r1}+d_{r2}}{2}, \tag{46}$$

$$a_4 = \frac{\sum d_{ai}}{4}, \tag{47}$$

$$\sin a_3 = \frac{d_{a2}+d_{a3}-d_{a1}-d_{a4}}{2\sqrt{d_1^2+\left(\frac{d_{a2}+d_{a3}-d_{a1}-d_{a4}}{2}\right)^2}}, \tag{48}$$

$$\cos a_3 = \frac{d_1}{\sqrt{d_1^2+\left(\frac{d_{a2}+d_{a3}-d_{a1}-d_{a4}}{2}\right)^2}}, \tag{49}$$

$$\sin a_2 = \frac{d_{r4}-d_{r1}}{\sqrt{d_2^2+(d_{r4}-d_{r1})^2}}, \tag{50}$$

$$\cos a_2 = \frac{d_2}{\sqrt{d_2^2+(d_{r4}-d_{r1})^2}}, \tag{51}$$

$$\sin a_5 = \frac{d_{a3}+d_{a4}-d_{a1}-d_{a2}}{2\sqrt{d_2^2+\left(\frac{d_{a3}+d_{a4}-d_{a1}-d_{a2}}{2}\right)^2}}, \tag{52}$$

$$\sin \alpha_5 = \frac{d_2}{\sqrt{d_2^2 + \left(\frac{d_{a3}+d_{a4}-d_{a1}-d_{a2}}{2}\right)^2}}. \quad (53)$$

The overall horizontal and vertical elastic displacements of translational joint can be now determined as function of horizontal and vertical elastic displacements of cam-followers by expressing the difference between two reference systems position and orientation ($O_1X_1Y_1Z_1$; $O_1X_2Y_2Z_2$) corresponding to non-deformed and respectively deformed joint by matrix [3].

6. NUMERICAL MODELS AND ANALYSIS

In order to accomplish a full analysis on the above mentioned standard Gantry robot (Gudel Fp4), the authors developed several applications and numerical models as it will be revealed in following chapters.

6.1. Custom-made application for determining the radial and axial loads on each cam-follower from robot's P-joints

The application was developed in Xcel by importing the mathematical model in former paper of the authors [6]. The results given by the application being the axial an radial loads on each cam-follower from robot's P-joints and also results regarding the estimated life of the most loaded guiding point.

In the Figs. 11 and 12 the screen captures from the above mentioned application are presented.

6.2. Numerical models developed for each cam-follower from robot's P-joints

In order to make an evaluation of the internal elastic behavior, specific for cam-followers that represent the guiding elements for gantry robot P-joints, the authors, by using a special made application for bearings assembles research (QBSA), developed numerical models for each guiding element. The results given by this application represent the internal elastic displacement for the studied cam-follower.

In Figs 13–15 the aspects from the above mentioned numerical models research can be seen.

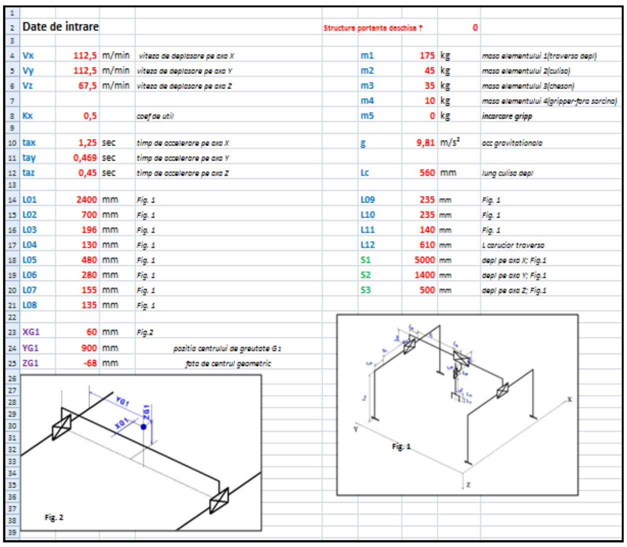


Fig. 11. Data introduction sheet.

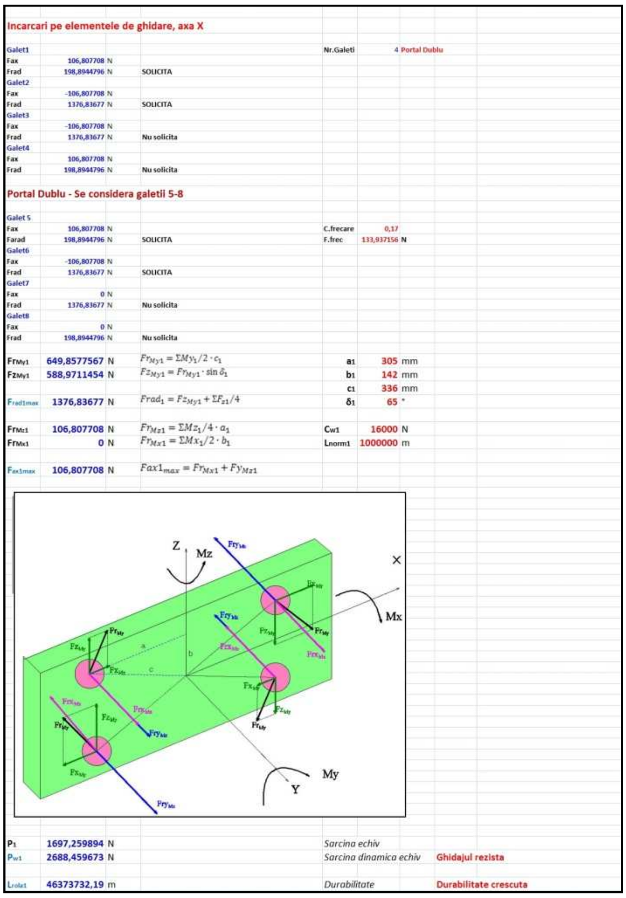


Fig. 12. Example of results sheet specific for a particular P-joint.

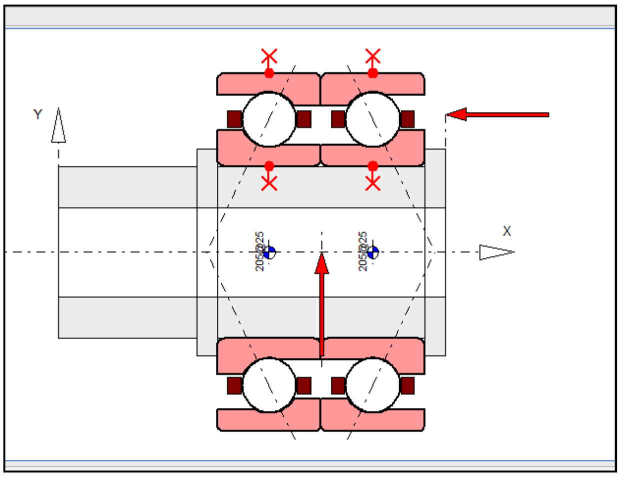


Fig. 13. Numerical model of a cam-follower.

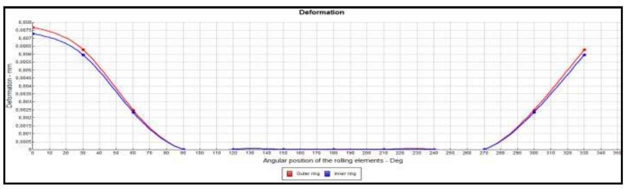


Fig. 14. Internal elastic deflection for a specific cam-follower.

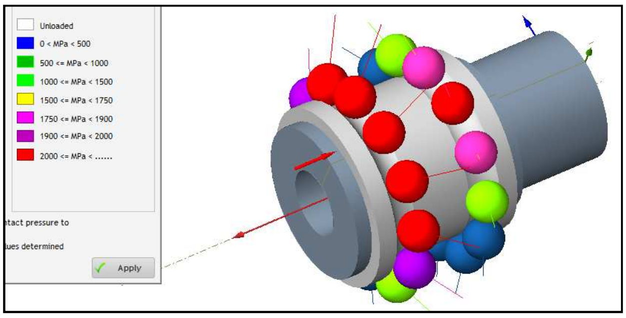


Fig. 15. Graphic representation of rolling elements loading corresponding to a specific cam-follower.

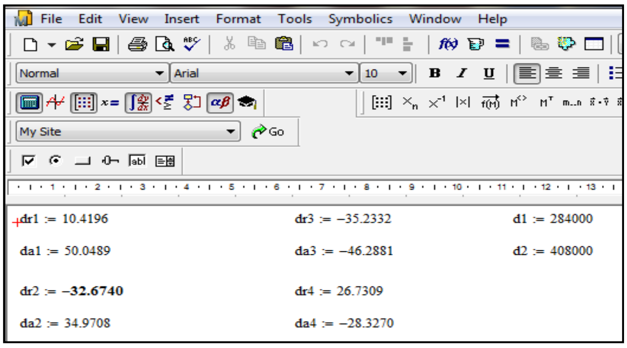


Fig. 16. Data entering sheet (axial and radial displacements resulted from using numerical model developed in QBSA).

$$(\Delta J)_2 := \begin{pmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oZ & aZ & pz \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.99999998006513 & -0.00019563087861 & 3.99786756736186 \times 10^{-5} & 0 \\ 3.99900557507744 \times 10^{-5} & 5.81676044223196 \times 10^{-5} & -0.99999999750866 & -11.1272 \\ 0.00019562855266 & 0.99999997917254 & 5.81754265524983 \times 10^{-5} & 2.60115 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Fig. 17. Example of displacement matrix generated for Gudel Fp4 Gantry robot *Y* axis subjected to particular loads.

6.3. Custom-made application for determining the total elastic displacement specific for a gantry robot P-joint

The above mentioned application were used by authors in order to obtain necessary results for the main purpose of the present paper – total elastic displacement of a gantry robot P-joint.

In order to have results regarding the total elastic displacement, the authors developed a custom-made application using MathCad functions in which the mathematical model presented in Chapter 5 was imported.

Figures 16 and 17 reveal examples of above mentioned application functionality.

7. CONCLUSIONS

The paper successively presents: the original mathematic modelling for Gantry IR overall elastic behaviour and specific algorithm / quantifying method of P joints elastic displacements influence on IR's volumetric accuracy (both of them generally valid for any kind of gantry robot and P-joint specific internal design), specific performances of an usual model of Güdel Gantry robot type and respectively it's virtual prototype already developed by authors for performing specific applied calculus related to this robot's P joints / overall elastic behaviour modelling, the mathematic model of overall gravitational and inertial load distribution, specific mathematic algorithm for loading behaviour evaluation related to robot's P-joints partially assemblies and each included cam following guiding components. In the first part of the paper were presented the general mathematical model, made for gantry robots, for determining the positioning error, valid for a real robot structure. Based on the Denavit-Hartenberg algorithm, an original mathematical model has been developed adding matrix expressions that are describing the errors induced by joint's specific kinematic model corresponding to *X*, *Y* and *Z* respectively robot axes, as well as specific errors of joints generated by elastic displacements, for the general case of gantry robots. A Gudel Gantry robot model virtual prototype has been developed too in order to perform applied calculus for a specific robot product type and as well to allow further approach based on FEM for robot's structural components elastic behavior evaluation.

In order to transform the mathematical model presented in the paper in a useful and practical tool, the authors developed several calculator applications and numerical models that comes as an addition of gantry robot elastic behavior studies.

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