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STRESS STATES IN PLANE AND RIBBED CIRCULAR PLATES

Carmen T. POPA[1,*], Radu I. IATAN[2], Ciprian I. MANESCU[3]

[1)] Lecturer, PhD, eng., Department of Food Engineering, University Valahia, Targoviste, Romania
[2)] Prof., PhD, eng., eng., University "Politehnica", Bucharest, Romania
[3)] Assist., PhD, eng., Inventiv SRL, Ploiesti, Romania

Abstract: *Circular or annular plates and mostly ribbed structures have multiple utilizations in numerous engineering domains: processing equipments, naval, metallurgy, civil constructions industries. The engineering ability, implemented in practice, introduced to reduce the consuming of materials, but also the energy consuming, alternative solutions: materials for replacement or rigidifying using ribs. The topic of this paper takes into discussion the particular case of the structures with constructive orthotropy.*
In this paper is presented the analysis of stresses states for a plane and for a ribbed plate, to observe the advantage of the rigidifying. The presence of the ribs leads to reduce the consuming of metallic material. The plates, of the same dimensions, are solicited at 0.3 MPa pressure. For comparison, we used the analytical, finite elements and experimental methods for the plane plate and for the ribbed plate, we used the finite elements and experimental methods. In the technical literature, for the ribbed plate there is an analytical method for calculating of the deformation and stresses states, only some approximations for particular cases. Finally, as we infer from the comparison of the results, we observe the advantage of the rigidity of the plate. In this case the value of the stresses lower about three times. Closed values are obtained using the variants of calculus: analytical, finite elements and experimental. However, we note that at the embedded plates the maximum of stresses is located on the contour.

Key words: *plane circular plate, circular plate with ribs, stresses states, numerical method, experimental method, analytical method.*

1. INTRODUCTION

The engineering ability, implemented in practice, introduced to reduce the consuming of materials, but also the energy consuming, alternative solutions: materials for replacement or rigidifying using ribs. The topic of this paper takes into discussion the particular case of the structures with constructive orthotropy.

Circular or annular plates [1], of constant or variable thickness, connected with rigidity elements-by casting, bonding or welding, are met in numerous engineering domains [2 and 3]. Radial and/or annular ribs can be disposed symmetrical or not, toward the middle surface of the proper plate.

The searches had been done to establish the displacements and stresses states at plates rigidified with ribs can be grouped in:

a) Approximate calculus methods of the stresses and displacements states;
b) Methods which abase the study at behaviour of the compound elements: plates and ribs, that are considered with different leaning ways;
c) Calculus methods that abase the structural orthotropic at material orthotropic;
d) Numerical methods [4–9];
e) Experimental methods [10].

In this paper the following aspects are realized:

- the determination of the stresses states for a plane plate, using the methods: analytical, finite elements and experimental;
- determination of the stresses states for a plane plate, using the methods: analytical, finite elements and experimental; the determination of the stresses states for a ribbed plate, using the methods: finite elements and experimental. The analytical method for the ribbed plate was analyzed in the scientific publication only for some particular cases;
- comparison of the obtained values.

2. PLANE PLATE ANALYZE

2.1. Elements finite method

We analyze the plane plate, having 32 holes, and geometrical characteristics from Fig. 1, using finite element method. We consider that the plate is fixed on circumference of the holes for the screws, which clamp this plate by the experimental recipient [13]. Using the program, the plate was divided in mesh-type tetrahedron elements. The plane plate with 32 holes were divided in a total of 16 696 elements and 5 561 nodes. We present the calculus variant with finite elements, the plate stresses distribution at 0.3 MPa being presented in Fig. 2. Stresses from Fig. 3 are obtained at pressure of 0.3 MPa,

* Corresponding author: University Valahia from Targoviste,
Tel.: 0744310647;
Fax: 035109907;
E-mail addresses: *carmenpopa2001l@yahoo.com* (C. Popa), *r_iatan@ yahoo.com* (R. Iatan), *cpmanescu@yahoo.com (C. Manescu*)

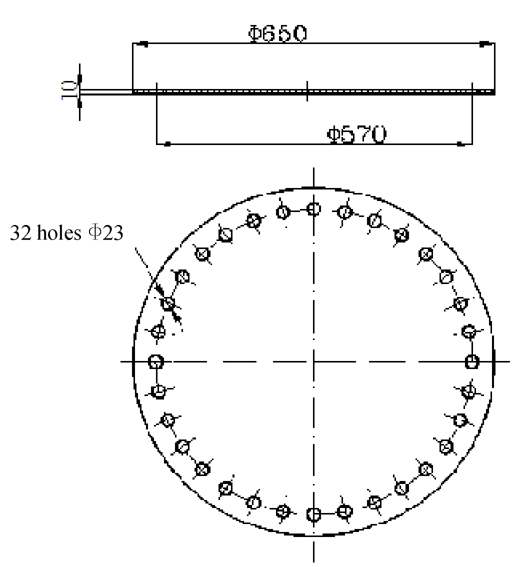

Fig. 1. Geometrical characteristics of the circular plane plate.

using a plan which passes through the centres of two exactly contrary holes, which are situated between two neighbour holes. Stresses values are given in Table 1.

2.2. Analytical method

In this case the plane plate is considered fixed on the line of the centres of the holes, at r_{cr}= 285 mm.

We use the relations (1, 2, and 3) for the evaluation of stresses on the circumference of leaning [11]:

$$\sigma_r = \pm 0{,}225 \cdot \frac{p \cdot r_{cr}^2}{\delta^2}\ ; \tag{1}$$

$$\sigma_\theta = \pm 0{,}75 \cdot \frac{p \cdot r_{cr}^2}{\delta^2}\ ; \tag{2}$$

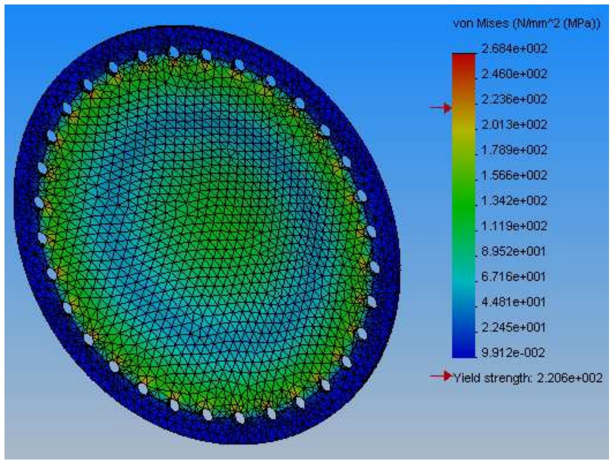

Fig. 2. Stresses of the fixed plane plate, at pressure 0.3 MPa.

$$\sigma_e(\mathrm{IV}) = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r \cdot \sigma_\theta}\ . \tag{3}$$

The radial and circumferential stresses at a certain radius are established with the relations:

$$\sigma_r = \pm \frac{6}{\delta^2} \cdot M_r\ ; \quad \sigma_\theta = \pm \frac{6}{\delta^2} \cdot M_\theta\ . \tag{4}$$

The expressions of the bending moments are:

$$M_r = \frac{p}{16} \cdot \left(1{,}3 \cdot r_{cr}^2 - 3{,}3 \cdot r^2\right); \tag{5}$$

$$M_\theta = \frac{p}{16} \cdot \left(1{,}3 \cdot r_{cr}^2 - 1{,}9 \cdot r^2\right). \tag{6}$$

where: $\sigma_r, \sigma_\theta, \sigma_e$ are radial, circumferential, equivalent stresses at median surface of the plate; p – uniform distributed pressure on the surface of the plate; r_{cr} –radius of the embedded circumference of the plate;

Table 1

Stresses σ [N/mm²], at 0.3 MPa pressure, for the fixed plane plate

r [mm]	**0**	**7**	**21**	**36**	**50**	**66**	**80**	**95**	**110**	**124**	**138**	**152**	**167**
σ [N/mm²]	129.8	129.7	128.3	125.4	120.9	113.5	108.1	99.88	91.58	81.19	72.56	62.68	59.79

178	**194**	**211**	**225**	**239**	**251**	**262**	**273**	**285**
57.99	62.67	76.43	95.4	113.2	127.3	159.3	244.4	44.31

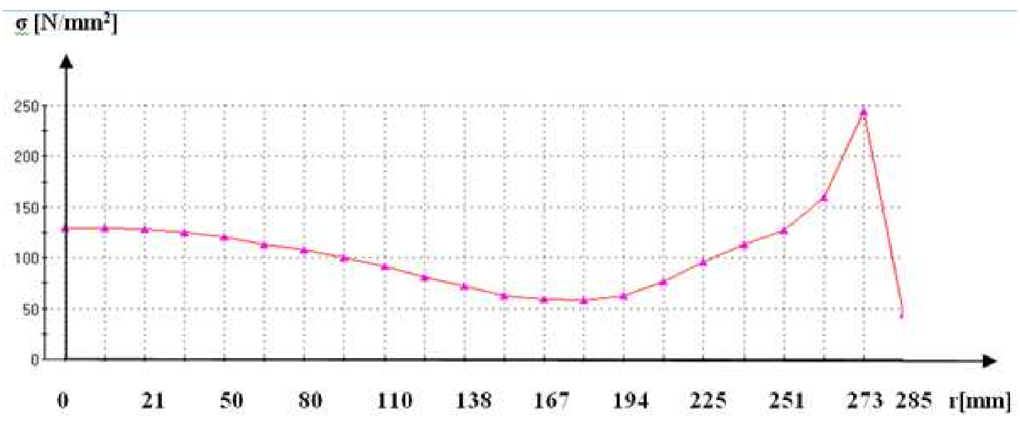

Fig. 3. Stresses of the fixed plane plate, at 0.3 MPa pressure.

Table 2

Stresses σ [N/mm2], at 0.3 MPa pressure, for the fixed plane plate, using finite elements and analytical methods

r[mm]	σ [N/mm^2]	
	Finite elements method	**Analytical method**
225	95.4	75

δ – thickness of the plate; M_r, M_θ – radial bending moment (acting in a diametral plane of the plane circular plate), respectively annular bending moment; r – current radius.

The value of equivalent stress on the leaning contour of the plate is $\sigma_{max\,2} = 162.4$ N/mm^2, for the pressure of 0.3 MPa. The equivalent stress at radius r = 225 mm (because at radius $r_{cr} = 285$ mm in MEF large increases of the stress are recorded) is σ_e = 75 N/mm^2.

As seen from Table 2, where the maximum values of stresses established by the two methods for calculating are processed, the values of stresses fond by finite element method are relatively close to those found by analytical method. In the case of finite element analysis, the plate had been fixed on the internal contour of the holes, and in the case of analytical method, the plate had been fixed in the continuous outline generated of the centre holes.

If it is supposed at a value of the pressure of 0.3 MPa, we obtain the maximal value of stress on the fixed line, like in Table 2.

2.3. Experimental method

In the experimental case, we take into account the measuring of specific linear deformations of the external points of the plate and calculus of the radial, circumferential and equivalent stresses, in the same conditions, using the strain gauges [5].

The components of the experimental stall are presented in Fig. 4, where: 1 – vessel with the plates for testing; 2 – manometer; 3 – branch; 4 – tap (open during the filling with water of the tap and closed during pressing); 5 – bend; 6 – pump group; 7 – funnel for the filling of the installation.

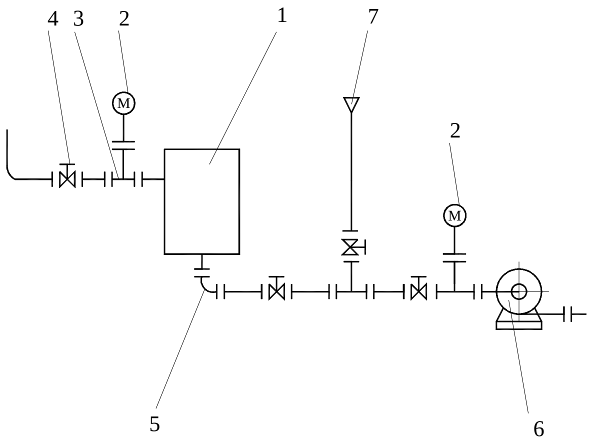

Fig. 4. Schema of the experimental stall.

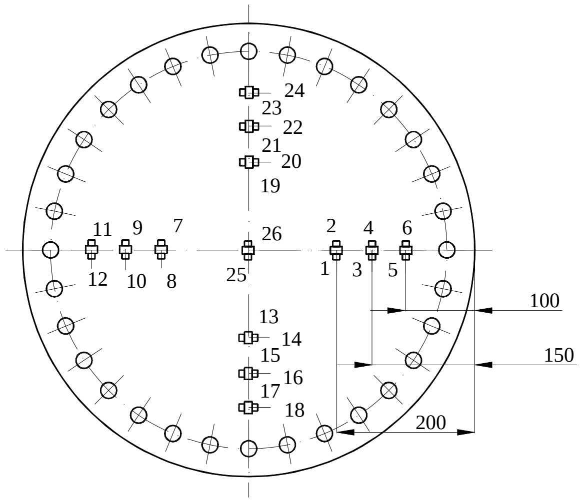

Fig. 5. The position of the strain gauges on the plane plate.

To assess the stresses developed both on the upper surface of plane plate and on the plate with radial ribs, under the action of the pressure created inside the experimental vessel, two perpendicular directions were accepted to be provided in the cross transducers, numbered with odd numbers (those in the direction radial) and with even numbers (transducers oriented in circumferential direction).

The zones where the strain gauges are fixed on the plate are presented in Fig. 5. The used strain gauges are KM120, with R = 120 Ω resistance and K = 2.04 ± 2 % elastic constant.

We measured the radial and circumferential deformations values, both at the increase and decrease of the pressure, obtaining the values: 0.05; 0.1; 0.15; 0.2; 0. 25; 0.3 MPa. We had been realized the processing of the experimental data using mathematical program, accepting a linear variation, which depends on parameter pressure. We obtained the equivalent stresses values, which are represented in the Table 3, using the fourth resistance criterion. On their basis, the diagram in Fig. 6 was obtained.

Used to calculate the radial, annular and equivalent stresses are the relations:

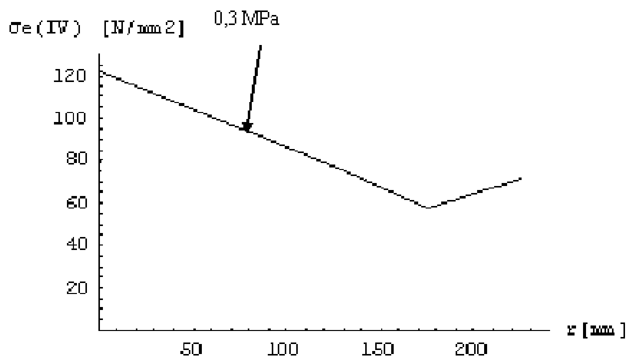

Fig. 6. The variation of the equivalent stresses which are calculated on the basis of the fourth resistance theory.

$$\sigma_r = \frac{E_p}{1-\nu_p^2}\cdot\left(\varepsilon_r + \nu_p \cdot \varepsilon_\theta\right); \qquad (7)$$

$$\sigma_\theta = \frac{E_p}{1-\nu_p^2}\cdot\left(\varepsilon_\theta + \nu_p \cdot \varepsilon_r\right); \qquad (8)$$

$$\sigma_e(IV) = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r \cdot \sigma_\theta}. \qquad (9)$$

Table 3

The values of the equivalent stresses at 0.3 MPa

p [MPa]	r [mm]	$\sigma_e\left[\mathrm{N/mm^2}\right]$			Obs.
		MA	MEF	ME	
0.3	0	118v8	130.1	121.3	$\sigma_e(\mathrm{IV})$
	125	76.1	83.8	77.2	$\sigma_e(\mathrm{IV})$
	175	51.0	66.7	58.2	$\sigma_e(\mathrm{IV})$
	225	75.0	74.4	70.7	$\sigma_e(\mathrm{IV})$

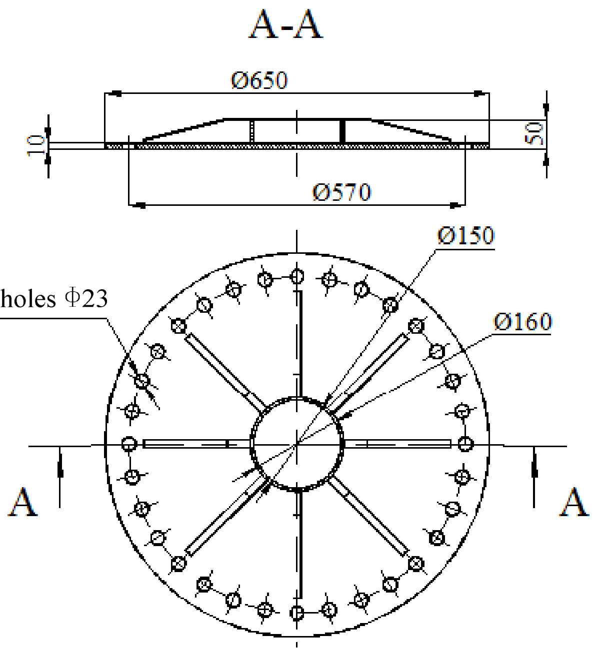

Fig. 7. Geometrical characteristics of the circular ribbed plate.

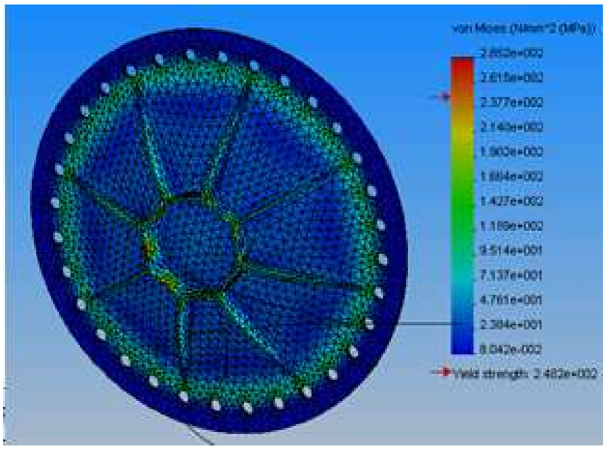

Fig. 8. Stresses of the fixed ribbed plate, at 0.3 MPa pressure.

Table 4

Stresses σ [N/mm²], at 0.3 MPa pressure, for the fixed plate with ribs

r [mm]	0	7	21	36	50	66	80	95	110	124	138	152	167
p = 0.3MPa	36.92	36.44	35.6	32.89	28.14	23.85	41.5	40.38	31.28	30.11	31.05	30.66	27.72

178	194	211	225	239	251	262	273	285
25.71	17.73	13.75	29.82	45.91	62.24	94.75	129.6	40.71

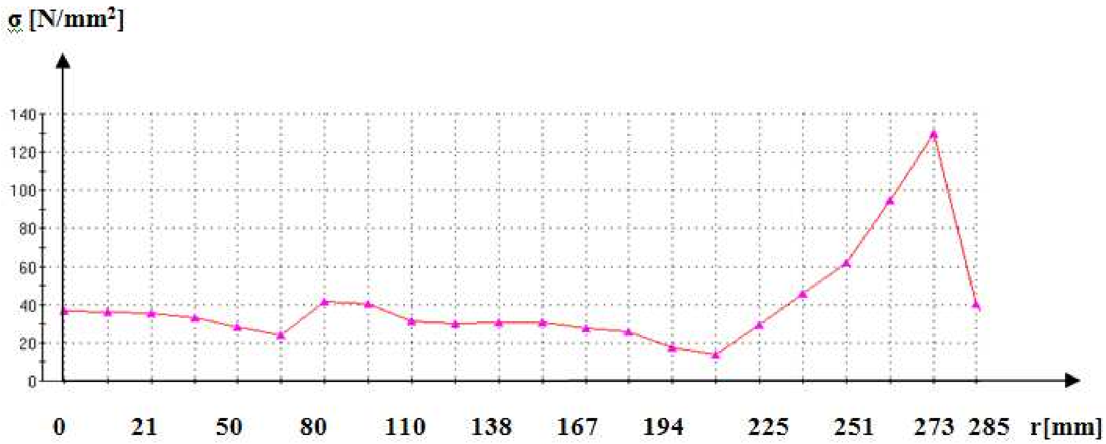

Fig. 9. Stresses of the fixed ribbed plate, at 0.3 MPa pressure.

3. RIBBED PLATE ANALYZE

3.1. Elements finite method

We analyze the ribbed plate, having 32 holes, and geometrical characteristics from Fig. 7, using finite element method. We consider that the plate is fixed on circumference of the holes for the screws, like in the case of the plane plate [13]. Also, we present the calculus variant with finite elements. The plate stresses distribution, at 0.3 MPa pressure is presented in Fig. 8.

Stresses from Fig. 9 are obtained at pressure 0.3 MPa, using a plan, which passes through the centres of two exactly contrary holes, which are situated between two neighbour holes. Stresses values are given in Table 4.

3.2. Experimental method

In the experimental case, we take into account the measuring of specific linear deformations of the external points of the plate and calculus of the radial, circumferential and equivalent stresses, in the same conditions, like in the case of the plane plate, using the strain gauges [13].

The zones where the strain gauges are fixed on the plate are presented in Fig. 10. We also used the strain gauges.

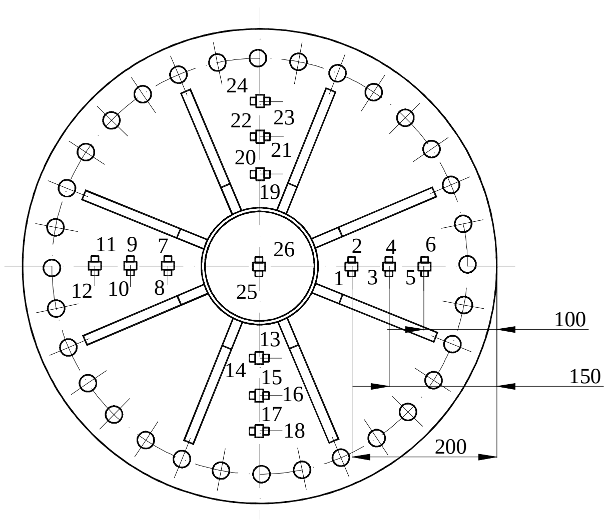

Fig. 10. The position of the strain gauges on the ribbed plate.

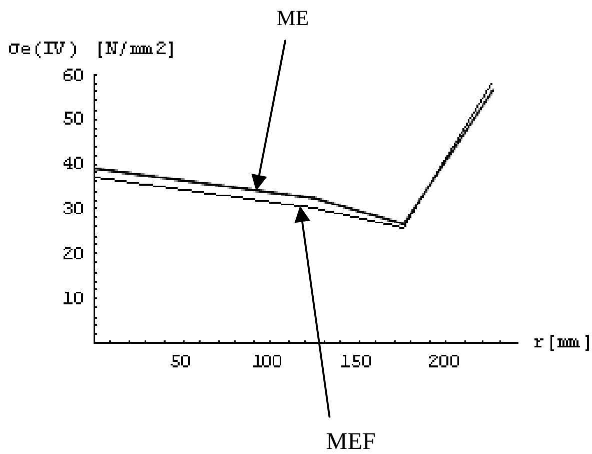

Fig. 11. The variation of the equivalent stresses which are calculated on the basis of the fourth resistance theory.

Table 5

The values of the equivalent stresses, which are theoretical and experimental calculated (MA), (MEF) and (ME), at 0.3 MPa pressure, for the ribbed plate

p [MPa]	r [mm]	$\sigma_e\,[\mathrm{N/mm^2}]$			Obs.
		MA	**MEF**	**ME**	
0.3	**0**	-	36.92	39.20	$\sigma_e(\mathrm{IV})$
	125	-	31.9	3,4	$\sigma_e(\mathrm{IV})$
	175	-	25.9	27.60	$\sigma_e(\mathrm{IV})$
	225	-	58.1	57,7	$\sigma_e(\mathrm{IV})$

We obtained the equivalent stresses values, which are represented in the Table 5, using the fourth resistance criterion. Figure 11 derived from these results.

4. CONCLUSIONS

We take into account the deformations, respectively stresses values, which are produced in different points of the plate. As we infer from the comparison of the results, we observe the advantage of the rigidity of the plate. In this case the value of the stresses lower about three times. Closed values are obtained using the variants of calculus: analytical, finite elements and experimental. However, we note that at the embedded plates the maximum of stresses is located on the contour.

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