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CYLINDRICAL SURFACES ENWRAPPING HELICAL SURFACES RACK-TOOL VERSUS PLANING TOOL

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Abstract: *Whirls of helical surfaces can be generated by enwrapping, within rolling method, by using rack-tools, or by planing, with tools having cylindrical surfaces. In this paper, we present comparatively two methodologies that can be used in profiling a cylindrical tool reciprocal enwrapped to a cylindrical helical surface with constant pitch: the Rack-tool method, for generating frontal profiles of the helical surface and the Contact line method, for profiling the cylindrical (planing) tool to generate helical slots. The identity between the two mentioned methods is proven by numerical examples. Therefore, the use of a simpler method (plane enwrapping) becomes possible to solve spatial enwrapping problems, and the computational volume substantially reduces, by transforming a 3-D enwrapping problem in a plane one. At the same time, the soft applications required by solving such a problem become simpler.*

Key words: *helical surface, enwrapping, rack-tool, planing tool, circular profile, helical slot.*

1. INTRODUCTION

The problematic of surfaces generation by enwrapping is diverse in the following cases:

- enwrapping generation approached by the method of rolling between ordinate whirls of surfaces (typical when machining gears teeth with involute profile);
- generation of helical surfaces by using cutting tools bounded by revolution surfaces (the disc-tool or the cylindrical tool);
- generation of surfaces having point-contact (the worm-tool case).

In all the above-mentioned situations, finding the shape of the reciprocal enwrapped surfaces – especially in cutting tools profiling (rack-tool, pinion cutter, worm-tool, disc-tool or planing tool), is based on Olivier fundamental laws, regarding enwrapping with linear (first degree) contact [1], or enwrapping with point-form (second degree) contact [2].

Moreover, theorems resulted from Olivier fundamental laws have been enounced – e.g. Gohman theorem [4, 5], or Nikolaev theorem (with application in the case of contact between a helical surface and a revolution surface) [2, 6].

Complementary theorems have also been developed in order to study the enwrapped surfaces:

- the Minimum distance method, with specifications regarding the case of enwrapping surfaces associated to a couple of rolling axodes (centrodes) [7];
- the Family of substituting circles method [7] conceived for profiling rack-tool or pinion cutter types of tools;
- the Plane generating trajectories method [8], with specific enouncements of the enveloping condition characterized by simplicity, and having applicability in the generation process of the ordinate whirls of surfaces (profiles) associated to a couple of rolling axodes (centrodes), or the helical surfaces (with disc-tools or with tools bounded by cylindrical surfaces.

By starting from the remarkable facilities offered by the new generation of graphical design environments (e.g. CATIA [3]), graphical methods dedicated to profile cutting tools have been recently developed. Their efficiency was already proven in profiling tools generating by rolling gears teeth flanks (rack-tools, pinion cutters, rotating cutters) [3], or tools with cylindrical surfaces (planing tools) for helical surfaces generation [3].

In this paper, we present two equivalent methods, which can be used to profile the rack-tool generating an ordinate whirl of helical surfaces, associated to a cylindrical revolution axode (in the case of cylindrical helical surfaces with constant pitch).

2. PROFILING OF THE RACK-TOOL USED TO GENERATE AN ORDINATE WHIRL OF HELICAL SURFACES

The ordinate whirl of helical surfaces, cylindrical and of constant pitch, associated to a cylindrical revolution axode, is presented, in principle, in Fig. 1, together to the co-ordinates systems to which the whirl of surfaces to be generated and the generator rack-tool surface are referred. The reference system meanings are:

- *XYZ* is a mobile system, attached to the whirl of surfaces to be generated;

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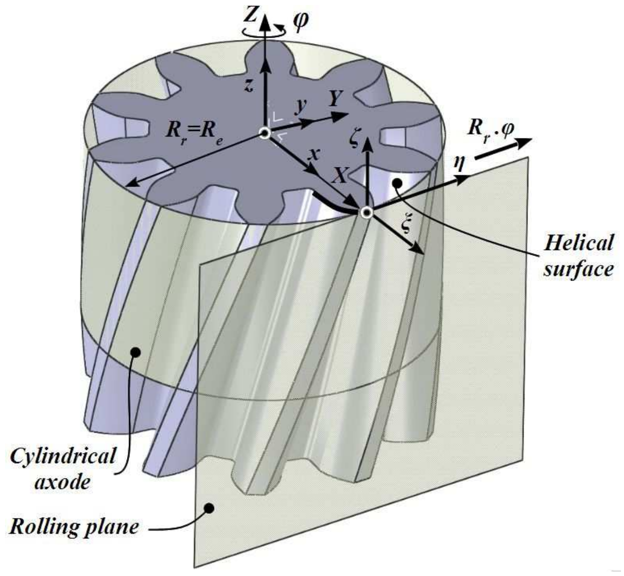

Fig. 1. The axode associated to the helical surfaces whirl, the rack-tool rolling plane and the reference systems.

- $\xi\eta\zeta$ – mobile system, attached to the generator rack-tool rolling plane;
- xyz – fix reference system.

The parametric equations of the helical surface, Σ, are defined relative to XYZ system:

$$\Sigma\left|\begin{array}{l} X = X(u,\varphi); \\ Y = Y(u,\varphi); \\ Z = Z(u,\varphi), \end{array}\right. \tag{1}$$

with u and φ meaning independent variable parameters.

The absolute motions of the reference systems associated to Σ surface and to the rack-tool are

$$x = \omega_3^T(\varphi_1)\cdot X, \tag{2}$$

respectively

$$x = \xi + a, \tag{3}$$

with

$$a = \begin{pmatrix} R_r \\ R_r\cdot\varphi_1 \\ 0 \end{pmatrix}. \tag{4}$$

In relation (4), R_r means the rolling radius of the cylindrical surface. The relative motion equation can be further expressed:

$$\xi = \omega_3^T(\varphi_1)\cdot X - a, \tag{5}$$

meaning the surfaces whirl motion, relative to the rack-tool, together to its inverse:

$$X = \omega_3(\varphi_1)[\xi + a]. \tag{6}$$

The generator rack-tool results as envelop of the family of surfaces Σ, in the relative motion referred to the rack-tool system:

$$(\Sigma)_{\varphi_1}\left|\begin{array}{l} \xi = X(u,\varphi)\cdot\cos\varphi_1 - Y(u,\varphi)\cdot\sin\varphi_1; \\ \eta = X(u,\varphi)\cdot\sin\varphi_1 + Y(u,\varphi)\cdot\cos\varphi_1; \\ \zeta = Z(u,\varphi). \end{array}\right. \tag{7}$$

The specific enveloping condition (Gohman) is

$$\overrightarrow{N_\Sigma}\cdot\overrightarrow{R_{\varphi_1}} = 0. \tag{8}$$

In relation (8) $\overrightarrow{N_\Sigma}$ represents the normal to Σ surface, which, in principle, results as:

$$\overrightarrow{N_\Sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dot{X}_u & \dot{Y}_u & \dot{Z}_u \\ \dot{X}_\varphi & \dot{Y}_\varphi & \dot{Z}_\varphi \end{vmatrix} = N_X\cdot\vec{i} + N_Y\cdot\vec{j} + N_Z\cdot\vec{k}. \tag{9}$$

Regarding the speed in the relative motion of the rack-tool relative to the reference system of the helical surfaces whirl, it has the expression:

$$R_{\varphi_1} = \frac{dX}{d\varphi_1} = \begin{pmatrix} Y(u,\varphi) + R_r\cdot\sin\varphi_1 \\ -X(u,\varphi) + R_r\cdot\cos\varphi_1 \\ 0 \end{pmatrix}, \tag{10}$$

where X has the significance from relation (6) or, written in vectorial form,

$$\overrightarrow{R_{\varphi_1}} = [Y(u,\varphi) + R_r\cdot\sin\varphi_1]\cdot\vec{i} + [-X(u,\varphi) + R_r\cdot\cos\varphi_1]\cdot\vec{j}. \tag{11}$$

The enveloping condition can be now rewritten (see relations (9) and (11)):

$$[Y(u,\varphi) + R_r\cdot\sin\varphi_1]\cdot N_X + [-X(u,\varphi) + R_r\cdot\cos\varphi_1]\cdot N_Y = 0 \tag{12}$$

This relation represents the enveloping condition between the helical surface Σ and the flank of the rack-tool to be profiled - S, in the relative motion between their two associated axodes: the revolution cylinder of R_r radius and the rolling plane, respectively.

The condition (12) means, in fact, a relation between Σ surface variable parameters, u and φ, and the kinematical parameter φ_1.

If parameter φ_1 value is kept constant, then the condition (12) is becoming a dependence relation, which can be written, for example, as

$$u = u(\varphi). \tag{13}$$

Such a dependence relation determines a locus onto the Σ surface, meaning the tangency curve between this surface and the rack-tool flank, S – the characteristic curve C_Σ.

The characteristic curve parametric equations, expressed in $\xi\eta\zeta$ reference system, look, in principle, as:

$$C_\Sigma\left|\begin{array}{l} \xi = \xi(u,\varphi); \\ \eta = \eta(u,\varphi); \\ \zeta = \zeta(u,\varphi); \\ u = u(\varphi). \end{array}\right. \tag{14}$$

3. PROFILING OF THE CYLINDRICAL SURFACE ENWRAPPED TO A CYLINDRICAL HELICAL SURFACE WITH CONSTANT PITCH, BY USING NIKOLAEV THEOREM

The generation of a helical surface with a planing tool – which executes a rectilinear motion, along the tangent to a helical line of the generated surface, is grounded, from geometrical point of view, on the contact between a helical surface and a cylindrical surface. The rectilinear generatrices of the cylindrical surface are parallel with the tangent to the helical line that usually belongs to the exterior cylinder of the surface to be generated, Fig. 2.

The parametric equations of the helical surface are defined into *XYX* reference system, having *Z* axis overlaid to the helical surface axis:

$$\Sigma\left|\begin{array}{l} X = X(u,\varphi); \\ Y = Y(u,\varphi); \\ Z = Z(u,\varphi), \end{array}\right. \tag{15}$$

with u and φ independent variable parameters.

If we denote by $\vec{t}$ the unitary vector of the tangent to the helical line corresponding to the cylinder of R_e radius – the exterior radius of the helical surface,

$$\vec{t} = \sin\beta\cdot\vec{j} + \cos\beta\cdot\vec{k}\ , \tag{16}$$

and by $\overrightarrow{N_\Sigma}$ the normal in the current point of Σ surface,

$$\overrightarrow{N_\Sigma} = N_X\cdot\vec{i} + N_Y\cdot\vec{j} + N_Z\cdot\vec{k}\,, \tag{17}$$

calculated with the help of a determinant, as indicated in (9), then the condition for finding the characteristic curve of contact between the helical surface Σ and the cylindrical surface having its generatrices oriented in $\vec{t}$ direction (16) is:

$$\overrightarrow{N_\Sigma}\cdot\vec{t} = 0\,. \tag{18}$$

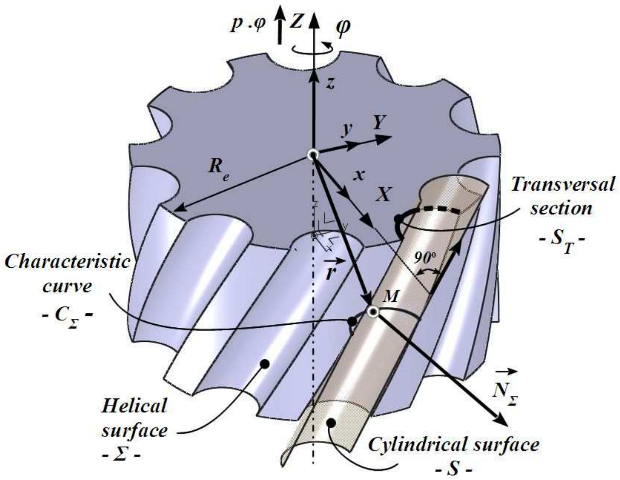

Fig. 2. The cylindrical and the helical surfaces, reciprocal enwrapped: reference systems, characteristic curve.

The condition (18), together to (16) and (17), gives an algebraic relation between the parameters u and φ, which further determines on Σ surface a locus – the characteristic curve C_Σ:

$$C_\Sigma\begin{cases}\Sigma\left|\begin{array}{l} X = X(u,\varphi); \\ Y = Y(u,\varphi); \\ Z = Z(u,\varphi); \end{array}\right. \\ u = u(\varphi), \end{cases} \tag{19}$$

or

$$C_\Sigma\left|\begin{array}{l} X = X(\varphi); \\ Y = Y(\varphi); \\ Z = Z(\varphi). \end{array}\right. \tag{20}$$

We are stating now that, under some conditions, in the case of a given surface Σ, *the characteristic curves* (14) *and* (20) *are coincident*.

We should notice that the value of β angle can be calculated after unrolling the cylinder of R_e radius, see also Fig. 3,

$$\tan\beta = \frac{R_e}{p}\,, \tag{21}$$

p meaning the helix parameter, calculated with

$$p = \frac{p_E}{2\cdot\pi}\,, \tag{22}$$

where p_E is the helix pitch.

Therefore, if the postulate from above proves to be true, then profiling the rack-tool used for generating an ordinate whirl of cylindrical helical surfaces, with constant pitch, can also be realized on the base of Nikolaev theorem (as source of condition (18)). Thus, we can approach this problem as a particular case of helical surfaces generation with tools bounded by revolution peripheral primary surfaces, having infinite radius – cylindrical surfaces.

4. NUMERICAL APPLICATIONS

We further develop numerical applications of the above-presented methods, for different types of cylindrical helical surfaces with constant pitch, aiming to exemplify the identity of characteristic curves found on the base of the two approaches.

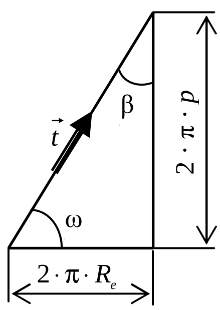

Fig. 3. The unrolled helix.

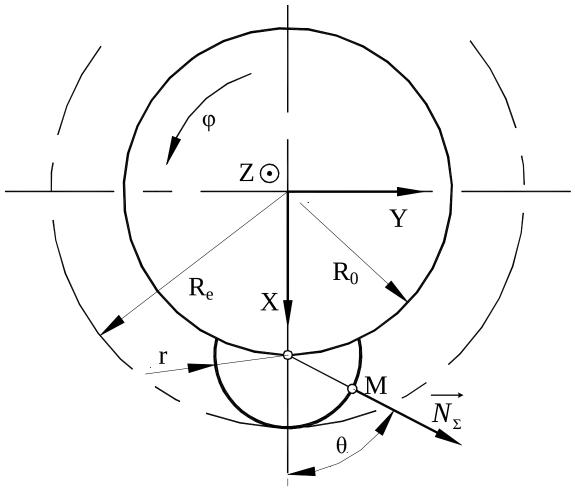

Fig. 4. The frontal section of the helical surface.

4.1. The helical surface with circular frontal profile

a. *Generation by rolling method*

The transversal profile of the helical surface is presented in Fig. 4. This is a profile specific to helical compressors rotors. The frontal section of the helical surface is defined relative to *XYZ* reference system. The co-ordinates of the current point *M*, on this profile are:

$$\begin{vmatrix} X = R_0 + r\cdot\cos\theta; \\ Y = r\cdot\sin\theta; \\ Z = 0, \end{vmatrix} \tag{23}$$

with θ variable parameter. Therefore, the equations of Σ helical surface of $\bar{Z}$ axis and *p* helical parameter are:

$$\Sigma\begin{vmatrix} X = R_0\cdot\cos\varphi + r\cdot\cos(\varphi+\theta); \\ Y = R_0\cdot\sin\varphi + r\cdot\sin(\varphi+\theta); \\ Z = p\cdot\varphi. \end{vmatrix} \tag{24}$$

In (24), the values of R_0, *r* and *p* are constructive parameters, while φ and θ are variable parameters. Hence, the condition to find the characteristic curve (see (14) together with notations from Fig. 1) becomes:

$$\begin{gathered}(r\cdot\sin\theta + R_e\cdot\sin\varphi)\cos\theta + \\ +(-R_0 - r\cdot\cos\theta + R_e\cdot\cos\varphi)\sin\theta = 0,\end{gathered} \tag{25}$$

or,

$$\sin(\varphi+\theta) = \frac{R_0}{R_e}\cdot\sin\theta. \tag{26}$$

The ensemble of (24) and (26) equations gives the surface of the reciprocal enwrapping rack-tool. The characteristic curve in the contact between the rack-tool flank and the helical surface is, then, determined by

$$S\begin{cases} X = R_0\cdot\cos\varphi + r\cdot\cos(\varphi+\theta); \\ Y = R_0\cdot\sin\varphi + r\cdot\sin(\varphi+\theta); \\ Z = p\cdot\varphi; \\ \varphi = \arcsin\left(\dfrac{R_0}{R_e}\right) - \theta, \end{cases} \tag{27}$$

when φ has a constant value.

In Table 1 and Fig. 5 one can see the co-ordinates of points from the characteristic curve, respective the shape of the rack-tool transversal section, whose profile is reciprocal enwrapped to the surface Σ defined by R_e = 35 mm, R_0 = 30 mm, *r* = 7 mm, *p* = 20 mm.

b. *Generation with planing tool* (*Nikolaev*)

In the case of the same helical surface (24), the shape of the cylindrical surface, reciprocal enwrapped, is found on the base of Nikolaev theorem (18). With notations from Fig. 3, the unitary vector $\vec{t}$ can be written as

$$\vec{t} = \cos\omega\cdot\vec{j} + \sin\omega\cdot\vec{k} \tag{28}$$

with

$$\tan\omega = \frac{p}{R_e}. \tag{29}$$

The normal to Σ helical surface, written in the current point, can be expressed by starting from the determinant

$$\overrightarrow{N_\Sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -R_0\sin\varphi - r\sin(\varphi+\theta) & R_0\cos\varphi + r\cos(\varphi+\theta) & p \\ -r\sin(\varphi+\theta) & r\cos(\varphi+\theta) & 0 \end{vmatrix} \tag{30}$$

Table 1

The characteristic curve (rolling method)

Crt. No.	X [mm]	Y[mm]	Z[mm]
1	30.000	7.000	0.000
2	30.094	6.932	-0.044
3	30.188	6.862	-0.089
4	30.281	6.792	-0.134
5	30.375	6.720	-0.179
6	30.468	6.647	-0.224
7	30.561	6.572	-0.269
8	30.654	6.496	-0.315
9	30.747	6.418	-0.360
10	30.839	6.339	-0.406
...	...	...	...
99	32.171	-11.048	-10.210
100	31.949	-11.445	-10.511
101	31.714	-11.846	-10.821

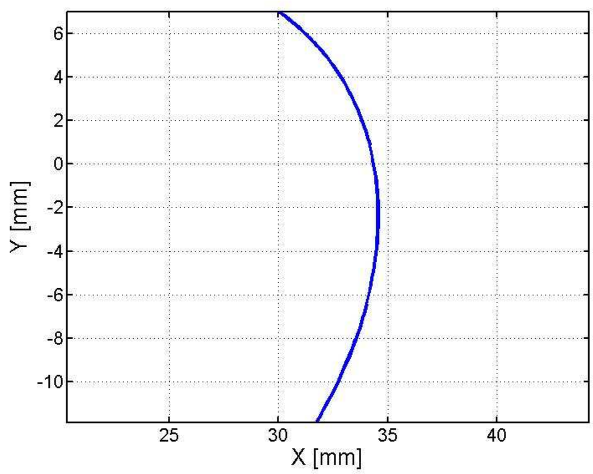

Fig. 5. The rack-tool transversal section.

Table 2

The characteristic curve (Nikolaev)

Crt. No.	X [mm]	Y[mm]	Z[mm]
1	29.996	6.983	–0.011
2	30.090	6.915	–0.055
3	30.184	6.845	–0.101
4	30.277	6.775	–0.145
5	30.371	6.703	–0.191
6	30.464	6.630	–0.236
7	30.557	6.556	–0.280
8	30.650	6.479	–0.326
9	30.742	6.402	–0.371
10	30.835	6.323	–0.417
...	...	...	...
99	32.151	–11.070	–10.232
100	31.929	–11.466	–10.533
101	31.693	–11.868	–10.844

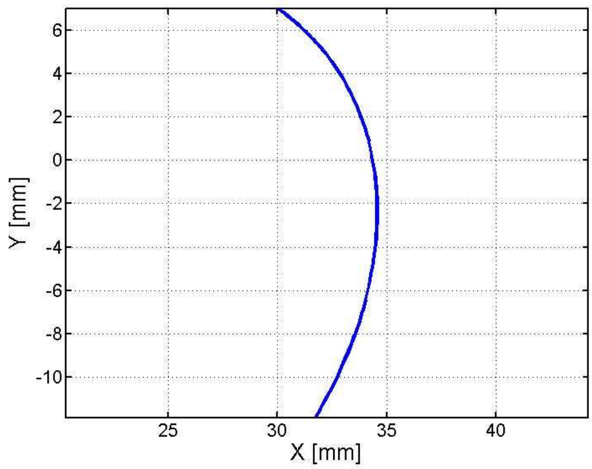

Fig. 6. The planing tool transversal section.

After calculus, from (30) we obtain the director parameters of $\overrightarrow{N_\Sigma}$ normal:

$$\begin{aligned} N_X &= -p \cdot \cos(\varphi + \theta); \\ N_Y &= -p \cdot \sin(\varphi + \theta); \\ N_Z &= R_0 \cdot \sin\theta. \end{aligned} \tag{31}$$

In Table 2 and Fig. 6 there are presented the co-ordinates of points from the characteristic curve, respective the shape of the cylindrical tool (planing tool) transversal section, obtained for the same values of Σ surface constructive parameters as above.

4.2. The helical surface with rectilinear frontal profile (helical slot)

The helical slots are used, for example, in mechanical structures like the gearboxes with sliding gears.

In Fig. 7 it is depicted the transversal profile of the helical surface of the mentioned type, which is referred to *XYZ* reference system, together to the constructive parameters defining its geometry. Therefore the rectilinear segment *AB* equations are:

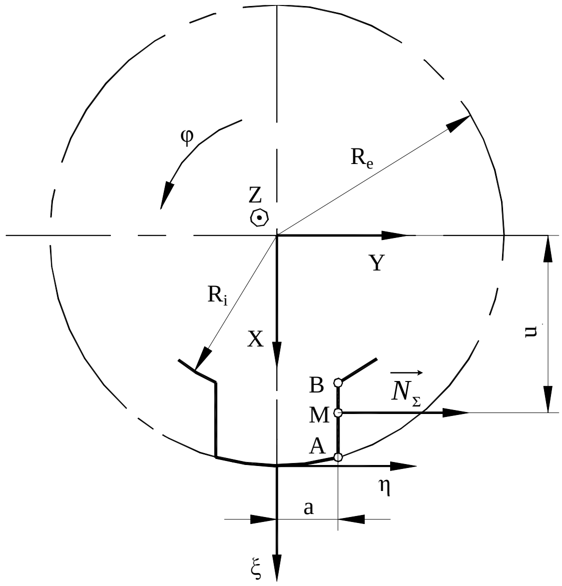

Fig. 7. The frontal section of the helical slot.

$$AB \left| \begin{aligned} X &= u; \\ Y &= a; \\ Z &= 0, \end{aligned} \right. \tag{32}$$

with *u* variable parameter. The equations of the frontal curves family result by considering the relative motion of *XYZ* system referred to ξηζ system:

$$(\Sigma)_\varphi \left| \begin{aligned} \xi &= u \cdot \cos\varphi - a \cdot \sin\varphi - R_e; \\ \eta &= u \cdot \sin\varphi + a \cdot \cos\varphi - R_e \cdot \varphi, \end{aligned} \right. \tag{33}$$

where φ means an angular, variable parameter.

a. *Generation by rolling method*

Rack-tool profiling in the transversal profile plane, by this method, requires determining the expressions for both $\overrightarrow{N_\Sigma}$ normal and $\overrightarrow{R_\varphi}$ vector. After calculus, we found:

$$\overrightarrow{N_\Sigma} = \vec{j}, \tag{34}$$

and

$$\overrightarrow{R_\varphi} = (a + R_e \cdot \sin\varphi)\vec{i} + (-u + R_e \cdot \cos\varphi)\vec{j}. \tag{35}$$

Therefore, from (8), (34) and (35) we obtain the enveloping condition in the final form

$$u = R_e \cdot \cos\varphi. \tag{36}$$

The ensemble of relations (33) and (36) gives the rack-tool profile reciprocal enwrapped to the helical slot:

$$\left| \begin{aligned} X &= R_e \cdot \cos^2\varphi - a \cdot \sin\varphi; \\ Y &= R_e \cdot \sin\varphi \cdot \cos\varphi + a \cdot \cos\varphi; \\ Z &= 0. \end{aligned} \right. \tag{37}$$

In Table 3 and Fig. 8 one can see the co-ordinates of points from the characteristic curve, respective the shape of the rack-tool transversal section, resulted when $R_e = 41$ mm, $a = 6$ mm, $p = 80$ mm.

Table 3

The characteristic curve (rolling method)

Crt. No.	*X* [mm]	*Y* [mm]	*Z* [mm]
1	33.734	8.919	41.929
2	33.809	8.876	41.731
3	33.884	8.833	41.532
4	33.959	8.790	41.333
5	34.034	8.747	41.132
6	34.109	8.705	40.931
7	34.184	8.663	40.729
8	34.259	8.621	40.526
9	34.334	8.580	40.322
10	34.409	8.538	40.117
...	...	...	...
99	40.894	6.038	13.030
100	40.948	6.029	12.406
101	41.000	6.021	11.749

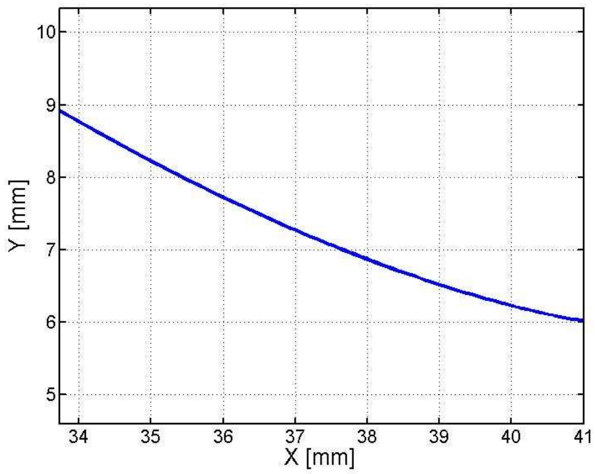

Fig. 8. The rack-tool transversal section.

Table 4

The characteristic curve (Nikolaev)

Crt. No.	*X* [mm]	*Y* [mm]	*Z* [mm]
1	33.746	8.912	41.896
2	33.822	8.868	41.695
3	33.898	8.824	41.494
4	33.974	8.781	41.293
5	34.049	8.739	41.092
6	34.124	8.697	40.890
7	34.198	8.655	40.689
8	34.273	8.614	40.488
9	34.347	8.573	40.287
10	34.421	8.532	40.086
...	...	...	...
99	40.904	6.036	12.918
100	40.960	6.027	12.264
101	41.010	6.020	11.611

b. *Generation with planing tool (Nikolaev)*

The normal to Σ surface, generated by giving to *AB* segment (32) a helical motion of *p* parameter, along *Z* axis, results from the determinant:

$$\overrightarrow{N_\Sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\varphi & \sin\varphi & 0 \\ -u\sin\varphi - a\cos\varphi & u\cos\varphi - a\sin\varphi & p \end{vmatrix}. \quad (38)$$

The unitary vector $\vec{t}$ has the same meaning and expression as above – see relations (28) and (29). The condition (18) becomes, in this particular case,

$$\left(p\sin\varphi\cdot\vec{i} - p\cos\varphi\cdot\vec{j} + u\cdot\vec{k}\right)\left(\cos\omega\cdot\vec{j} + \sin\omega\cdot\vec{k}\right) = 0 \quad (39)$$

In Table 4, one can see the co-ordinates of characteristic curve points, obtained for the same values of Σ surface constructive parameters as in section 4.2-a. The planing tool transversal section resulted almost the same with the rack-tool one (see Fig. 8), so we did not represent it any more.

5. CONCLUSIONS

The analysis of the contact between a helical and a cylindrical surface was realized by using two distinct enveloping principles: *i)* generation of the rack-tool enwrapped to the helical surface frontal profile, by rolling method, and *ii)* generation with a planing tool, by using Nikolaev theorem – specific to helical surfaces generation with tools bounded by revolution surfaces.

The numerical applications sustain the conclusion that both cylindrical tools (rack-tool and planing tool), generating the same surface, have identical profiles. Therefore, profiling of the tool that generates complex helical surfaces (compressor rotors) by planing can be successfully done by using the simpler principle of generating with a rack-tool – the plane case. Hence, the computational volume substantially reduces, by transforming a 3-D enwrapping problem in a plane one. At the same time, the soft applications required by solving such a problem become simpler.

REFERENCES

[1] F.L. Litvin, *Theory of gearing*, NASA, Scientific and Technical Information Division, Washington D.C., 1984.

[2] V.S. Liukshin, *Theory of screw surfaces in cutting tool design*, Machinostroyenie, Moscow, 1968.

[3] S. Berbinschi , V. Teodor, N. Oancea, *3D graphical method for profiling tools that generate helical surfaces*, Int J Adv Manuf Technol, Vol. 60, 2012, pp. 505–512.

[4] G. Nankov, V. Ivanov, *CAD Oriented Mathematical Model for Profiling of Tools For Machining of Helical Surfaces*, International Science Conference AMTECH'95 Proceedings, Ruse, Bulgaria, section 4, 1995, pp. 79–93.

[5] S.P. Radzevich, *Kinematics Geometry of Surface Machining*, CRC Press, London, 2008.

[6] N. Oancea, *Generarea suprafeţelor prin înfăşurare, vol I* (Surface Generation Through Winding), "Dunărea de Jos" Publishing House, 2004.

[7] N. Oancea, *Generarea suprafeţelor prin înfăşurare, vol II* (Surface Generation Through Winding), "Dunărea de Jos" Publishing House, 2004.

[8] V. Teodor, *Contributions to the Elaboration of a Method for Profiling Tools*, Lambert Academic Publishing, 2010.