# STUDYING THE BEHAVIOR OF CNC LASER CUTTING MACHINES IN THE CONTOURING REGIME 

Radu - Eugen BREAZ ${ }^{1, *}$, Cristina-Maria BIRIS ${ }^{2}$, Octavian Constantin BOLOGA ${ }^{3}$, Sorin Tîrnovean ${ }^{4}$<br>${ }^{1,3)}$ Prof., PhD, Department of Industrial Machines and Equipments, "Lucian Blaga" University of Sibiu, Sibiu, Romania<br>${ }^{2)}$ Ass. Prof., PhD, Department of Industrial Machines and Equipments, "Lucian Blaga" University of Sibiu, Sibiu, Romania<br>${ }^{4)} \mathrm{PhD}$ Student, Department of Industrial Machines and Equipments, "Lucian Blaga" University of Sibiu, Sibiu, Romania


#### Abstract

The paper presents a set of modeling and simulation tools which were developed by the authors in order to allow the users of CNC laser cutting machines to study and improve the machine dynamics and accuracy. The approach is based upon a Matlab \& Simulink model of the feed drives of the machine, but it also takes into consideration the kinematic inputs (position, velocity and acceleration) and, furthermore, some experimental work was performed. The main target of this approach was to offer a set of straight-forward, yet reliable solutions which may be implemented at the workshop level. A mathematical model of the circular interpolation process was implemented in Matlab \& Simulink software package and a diagram for generating the reference inputs was built and integrated in the general model of the feed drives of the machine. The numerical character of generating the kinematic inputs was also introduced in the model, by means of the sampling period. DC servomotors were considered as actuation devices for the feed drives. Experimental researches were also performed in order to validate the proposed set of tools. A number of parts were processed on an industrial laser cutting machine, using the proposed control strategies and their accuracy was assessed.


Key words: circular interpolation; contouring; CNC machine-tools; modeling; motion control; simulation.

## 1. INTRODUCTION

Parts' processing by means of laser cutting is characterized by high toolpaths velocities, reduced quantity of waste, high accuracies and quality of the manufactured parts [1-4]. In comparison with oxy-gas and plasma jet cutting technologies, the laser cutting process leads to a narrower heat-affected zone and a smaller kerf width [5-8].

The most common way 2D laser cutting technology is implemented in industrial systems is the CNC (computer numerically controlled) profiling machine. Although the prices of such machine are still high, their use is wide-spread at the small workshops level. The control system of these machines is a typical 2-axis CNC system, with simultaneous coordinated movements in $X Y$ plane. The $Z$ axis is also numerically controlled, but simultaneous movements on 3-axis are seldom programmed. The laser cutting machines motion control architecture is quite similar to the control structure of the CNC milling machines and lathes. Typical CNC motion control equipment is based upon rotational servomotors, position and speed transducers (such as linear/rotational encoders and tachometers) and ball screw transmissions.

[^0]However, there are some important particularities between CNC laser cutting machines and CNC milling machines or lathes. For example, technological forces (cutting forces and friction forces) are significantly smaller and the feeds are significantly higher within the process of laser cutting.

The motion control system uses advanced servocontrol algorithms and techniques (PID control, crosscoupling control, real-time error compensation) in order to minimize the tracking and contouring errors [9-15]. Most of the above-mentioned solutions either have to be applied in the designing phase of the machine tools or require cumbersome alterations inside the CNC controller.

Taking the above mentioned facts into consideration, there is still great interest between the users for a better understanding of the machine behaviour and for finding efficient ways to improve both its dynamics and accuracy.

However, at the workshop level there is neither time nor money for doing complex experimental research or for modifying the inner structure of the CNC controller.

The work presented here intends to offer the user a set of simulation diagrams which may be used for studying the behaviour of the laser cutting machine in the circular interpolation regime, which is one of the most demanding by the point of view of motion control.

The proposed tools are intended to be used at the workshop level, offering the user a straight-forward method for improving the machine-tools dynamic behaviour and manufacturing accuracy.

## 2. CIRCULAR INTERPOLATION

Reference word circular interpolation method involves the issue of reference position inputs on both axes as binary words, at time intervals equal with the CNC equipment sampling time.

For generating a circular trajectory in XOY plane, by means of a circular interpolation algorithm, the main requirement is to held the velocity on the path, F , constant. Consequently, the velocities on $O X$ and $O Y$ axes, $F_{x}$ respectively $F_{y}$ may be expressed as:

$$
\left\{\begin{array}{l}
F_{x}=F \sin \theta(t)  \tag{1}\\
F_{y}=F \cos \theta(t)
\end{array} .\right.
$$

where:
$\theta(t)$ - angle at moment $t$ (Fig. 1).
The relationship between the angle $\theta(t)$, the path velocity, the angular speed and the circle radius may be expressed by the following equations:

$$
\begin{equation*}
\theta(t)=\frac{F \cdot t}{R} . \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
F=\omega \cdot R=\frac{2 \pi}{T} \cdot R \tag{3}
\end{equation*}
$$

where:
R - radius of the circle [m];
$T$ - time for generating a whole circle (period) [s].
$\omega$ - angular speed [rad/s].
The circular trajectory is usually approximated by small linear segments. The number of these segments has to be optimized in such way that the deviation from the circular trajectory is smaller than BLU (the basic length unit of the CNC equipment), which is usually $1 \mu \mathrm{~m}$.

Equation (3) leads to:

$$
\begin{equation*}
\omega=\frac{2 \pi}{T}=\frac{F}{R} . \tag{4}
\end{equation*}
$$



Fig. 1. Reference word circular interpolation.

Figure 1 shows the relationship between two successive interpolated points for the reference word interpolator for a circular arc. Based on Fig. 1, the following equations may be written:

$$
\left\{\begin{array}{l}
\cos \theta(i+1)=A \cos \theta(i)-B \sin \theta(i)  \tag{5}\\
\sin \theta(i+1)=A \sin \theta(i)+B \cos \theta(i)
\end{array}\right.
$$

where

$$
\begin{align*}
& A=\cos \alpha, \quad B=\sin \alpha \\
& \theta(i+1)=\theta(i)+\alpha \tag{6}
\end{align*}
$$

and
$\alpha$ - angular displacement increment of the interpolation algorithm (angular step of the interpolation algorithm [degrees].

The coordinates of the final point of the linear segment used for the approximation, $x(i+1), y(i+1)$ may be expressed with the equation:

$$
\left\{\begin{array}{l}
x(i+1)=R \cos \theta(i+1)  \tag{7}\\
y(i+1)=R \sin \theta(i+1)
\end{array}\right.
$$

From (5) and (6), relationships which allow the calculation of the current point $i+1$, based upon the coordinates of the previous point $i$ may be written:

$$
\left\{\begin{array}{l}
x(i+1)=A x(i)-B y(i)  \tag{8}\\
y(i+1)=A y(i)+B y(i)
\end{array} .\right.
$$

The relationship between the linear displacement increments on each axis, $\Delta x(i), \Delta y(i)$ and the coordinates of the points with the linear segment may be expressed as:

$$
\left\{\begin{array}{l}
\Delta x(i)=x(i+1)-x(i)=(A-1) x(i)-B y(i)  \tag{9}\\
\Delta y(i)=y(i+1)-y(i)=(A-1) y(i)+B x(i)
\end{array} .\right.
$$

The velocities on each axis $F_{x}(i), F_{y}(i)$, may be expressed as:

$$
\left\{\begin{array}{l}
F_{x}(i)=F \frac{\Delta x(i)}{\Delta s(i)}  \tag{10}\\
F_{y}(i)=F \frac{\Delta y(i)}{\Delta s(i)}
\end{array}\right.
$$

where:

$$
\begin{equation*}
\Delta s(i)=\sqrt{(\Delta x(i))^{2}+(\Delta y(i))^{2}} \tag{11}
\end{equation*}
$$

By combining (5) and (7), the following equation may be written:

$$
\left\{\begin{array}{l}
x(i+1)=R[A \cos \theta(i)-B \sin \theta(i)]  \tag{12}\\
y(i+1)=R[A \sin \theta(i)+B \cos \theta(i)]
\end{array}\right.
$$

Taking into consideration (2) and the numerical character of the interpolation algorithm, the following equation may be written for the time step $T_{i}$ :

$$
\begin{equation*}
\theta(i)=\frac{F}{R} T_{i} . \tag{13}
\end{equation*}
$$

The following equation may be also written:

$$
\begin{equation*}
T_{i}=i \cdot T_{s} . \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
T=n \cdot T_{s} . \tag{15}
\end{equation*}
$$

where
$T_{\mathrm{s}}$ - sampling period [s];
$k$ - number of samples at the moment $T_{k}$;
$n$ - total number of samples.
Consequently (12) may be rewritten as:

$$
\left\{\begin{array}{l}
x(i+1)=R\left[A \cos \left(2 \pi \cdot \frac{i}{n}\right)-B \sin \left(2 \pi \cdot \frac{i}{n}\right)\right]  \tag{16}\\
y(i+1)=R\left[A \sin \left(2 \pi \cdot \frac{i}{n}\right)+B \cos \left(2 \pi \cdot \frac{i}{n}\right)\right]
\end{array} .\right.
$$

The total number of samples may be calculated by combining (3) and (15) as:

$$
\begin{equation*}
n=\frac{2 \pi \cdot R}{F \cdot T_{s}} \tag{17}
\end{equation*}
$$

Consequently, (16) may be rewritten as:

$$
\left\{\begin{array}{l}
x(i+1)=R\left[A \cos \left(\frac{F \cdot T_{s}}{R} \cdot i\right)-B \sin \left(\frac{F \cdot T_{s}}{R} \cdot i\right)\right]  \tag{18}\\
y(i+1)=R\left[A \sin \left(\frac{F \cdot T_{s}}{R} \cdot i\right)+B \cos \left(\frac{F \cdot T_{s}}{R} \cdot i\right)\right]
\end{array}\right.
$$

The angular displacement increment $\theta_{\text {incr }}$ may be calculated as:

$$
\begin{equation*}
\theta_{\text {incr }}=\frac{2 \pi}{T} \cdot T_{s}=\frac{F}{R} \cdot T_{s} . \tag{19}
\end{equation*}
$$

and the linear displacement increment $l_{\text {incr }}$ as

$$
\begin{equation*}
l_{\text {incr }}=2 R \cdot \frac{\theta_{\text {incr }}}{2 \pi}=\frac{F}{\pi} T_{s} \tag{20}
\end{equation*}
$$

Equation (20) shows the relationship between the linear displacement increment, velocity on the path and the sampling period.

The minimum angular displacement of the $X$ and $Y$ feed drives $\beta$ may be calculated as:

$$
\begin{equation*}
\beta=\frac{360^{0}}{N_{i m p}} \tag{21}
\end{equation*}
$$

where
$N_{\text {imp }}$ - number of pulses emitted by the incremental encoder at a full rotation.

The minimum linear displacement of the $X$ and $Y$ feed drives, BLU, may be calculated as:

$$
\begin{equation*}
B L U=\frac{\beta \cdot p_{s}}{360^{0}} . \tag{22}
\end{equation*}
$$

where
$p_{s}-$ step of the leadscrew.
Considering the linear displacement increment lincr, calculated according to (20) being equal with BLU, the following equation may be written:

$$
\begin{equation*}
B L U=\frac{F}{\pi} T_{s} . \tag{23}
\end{equation*}
$$

From (23) the sampling period $T_{\mathrm{s}}$ may be expressed as:

$$
\begin{equation*}
T_{s}=\frac{B L U \cdot \pi}{F} . \tag{24}
\end{equation*}
$$

Equation (24) shows the fact that the sampling period $T_{s}$ depends on the path velocity $F$.

## 2. SIMULATION

In order to simulate the interpolation algorithm, a simulation diagram, under Matlab \& Simulink was built (Fig. 2). The simulation diagram from Fig. 2 basically uses sample based sine wave blocks.

The simulation diagram from Fig, 2 was implemented in a more complex simulation diagram which models the behaviour of a two-axis ( $X Y$ ) motion control system within the structure of CNC laser cutting machine. The overall simulation diagram is presented in Fig. 3.

This research is based upon a model of a feed drive, using a DC servomotor as actuation device [16-17]. This kind of motor is widely used in the design of CNC laser cutting machines by a large number of manufacturers. The simulation diagram for the motion control on $X$ axis is presented in Fig. 4.


Fig. 2. Simulation diagram for the interpolation algorithm.

The input data was gathered taking into consideration the experimental system, a feed drive of a CNC laser cutting machine.

The position control systems, on all feed dives uses as actuation device $\alpha 6 / 3000$ GE Fanuc DC brushed servomotors [18].

The significance and the specific data for the simulation diagram form Fig. 4 are:
$R$ - motor armature winding resistance ( $0.18 \Omega$ );
$J$ - rotor and load inertia ( $0.26 \cdot 10^{-2} \mathrm{kgm}^{2}$ );
$B-$ viscous friction constant ( $0.75 \cdot 10^{-4} \mathrm{Nms} / \mathrm{rad}$ );
$K_{t}-$ motor torque constant ( $0.6 \mathrm{Nm} / \mathrm{A}$ );
$K_{v}$ - velocity constant ( $0.2 \mathrm{~V} \mathrm{~s} / \mathrm{rad}$ );
$K_{a}$ - amplifier gain (8.557);
$K_{t h}$ - tachometer gain ( $0.007 \mathrm{Vs} / \mathrm{rad}$ );
$K_{p x}-X$ axis position controller proportional gain (2730.7 bit/m);
$K_{d x}-X$ axis position controller derivative gain (0);
$K_{f f x}-X$ axis position controller feed-forward gain (0);
$K_{c}$ - digital to analog converter gain ( $0.000144 \mathrm{~V} / \mathrm{bit}$ );
$K_{e}$ - encoder gain ( $2500 / 2 \pi$ pulses $/ \mathrm{rad}$ );
$K_{g}$ - gain of the mechanical actuator ( $0.01 / 2 \pi \mathrm{~m} / \mathrm{rad}$ ).
The values of the position controller proportional gains on $X$ and $Y$ axis may be expressed by:


Fig. 3. Simulation diagram for the XY motion control system.


Fig. 4. Simulation diagram for the feed drive ( X axis).

$$
\begin{equation*}
v_{c m d}=e_{f o l} \cdot\left(K_{p x(y)} \cdot K_{c}\right) . \tag{25}
\end{equation*}
$$

where:
$v_{c m d}$ - velocity command or [V];
$e_{\text {fol }}$ - following or tracking error [m].
Velocity command ( $v_{c m d}$ ) represents the analog output corresponding to a feederate resulting in 1 mm of following error. It may be expressed also as:

$$
\begin{equation*}
v_{c m d}=\frac{F \cdot U_{\max }}{F_{\max }} \tag{26}
\end{equation*}
$$

where:
$F$ - working feedrate [ $\mathrm{mm} / \mathrm{min}$ ];
$F_{\text {max }}$ - maximum feedrate [ $\mathrm{mm} / \mathrm{min}$ ].
$U_{\text {max }}$ - maximum voltage output of the digital-to-analog converter [V]

For this particular application (corresponding to the value indicated in the machine documentation), $U_{\max }$ was set to 10 V . Taking into consideration following error of 1 mm , we reach to:

$$
\begin{equation*}
K_{p x(y)}=\frac{F \cdot U_{\max }}{F_{\max }} \cdot \frac{1}{K_{c}} . \tag{27}
\end{equation*}
$$

Figure 5 shows a detail from a simulated reference and actual trajectory for processing a full circle. For the initial set of data, a difference of $10 \mu \mathrm{~m}$ on $X$ axis and $27 \mu \mathrm{~m}$ on $Y$ axis between the two trajectories may be observed.

One of the approaches recommended in the literature, in order to compensate such type of errors is to increase the position controller proportional gain on the $Y$ axis.

Figure 6 shows a detail from a simulated reference and actual trajectory for a full circle, using a $K_{p y}=$ $1.5 \cdot K_{p x}$. From Fig. 6 it may be noticed that both differences between the two trajectories, on $X$ and $Y$ axis, are around $7 \mu \mathrm{~m}$.


Fig. 5. Simulated current and reference trajectory, $K_{p x}=K_{p y}$ (detail).


Fig. 6. Simulated current and reference trajectory, $K_{p y}=1.5 \cdot K_{p x}$ (detail).

## 4. EXPERIMENTAL RESEARCH

In order to validate the proposed tools, some experimental researches were performed on a CNC laser cutting machine. The part presented in Fig. 7 was processed and some of the dimensional characteristics were measured on a coordinate measuring machine (CMM). A working feedrate $F=2000 \mathrm{~mm} / \mathrm{min}$ was used, and the maximum feedrate taken into consideration was $F_{\text {max }}=10000 \mathrm{~mm} / \mathrm{min}$. The dimensions of the parts, marked with a small hexagon on Fig. 7 were measured and compared with their reference values.

However, for the circular dimensions, a somehow different procedure was used. The $\varnothing 44 \mathrm{~mm}$ circle noted with " 1 " was scanned and an igs "points cloud" file was created (Fig. 8). The points cloud contains geometrical information regarding the coordinates of the points from the processed circle, but the date has to be extracted using specialized software.

Consequently, the coordinates of the points were extracted and saved in a data file, which was after that sent to Matlab software package for further processing and display. The process was repeated three times, for different values of the position controller proportional gain: $K_{p y}=K_{p x}$ (initial approach), $K_{p y}=1.2 \cdot K_{p x}$, and $K_{p y}=1.5 \cdot K_{p x}$.


Fig. 7. Processed part.


Fig. 8. Point cloud file.


Fig. 9. Processed circular trajectories (detail, first quadrant).

The results are presented in Fig. 9 and Fig. 10, where the reference trajectory ( $\varnothing 44 \mathrm{~mm}$ circle) and the trajectories obtained by cutting the parts are compared.

It may be observed that the most accurate trajectory is the one which used $K_{p y}=1.5 \cdot K_{p x}$ for the $X$ and $Y$ axis position controller proportional gains. These facts validate both the proposed approach of increasing the proportional gain on $X$ axis and the modeling and simulation tools.


Fig. 10. Processed circular trajectories (detail, second quadrant).

## 5. CONCLUSION

The authors proposed a straight-forward approach for modeling and simulating two-axis motion control system of an industrial CNC laser cutting machine, which is nowadays one of the most common industrial equipment used for metal cutting.

In order to allow the user of the CNC machine to perform a study about the influence of the working regime, the technological and the control parameters within the CNC equipment upon the behaviour of the machine and the accuracy of the parts, a set of simulation diagrams under Matlab \& Simulink software package were developed.

First, a diagram for simulating reference word circular interpolation algorithm was presented. The interpolation process is used for generating kinematic inputs for simultaneous movements on two axes. The algorithm and consequently the simulation diagram were modified and developed in order to take into consideration specific parameters of the machine and the cutting process, such as the working feedrate and the sampling period of the CNC equipment.

The interpolation algorithm was implemented into a more complex diagram for simulating the behaviour of an XY motion control system within a CNC laser cutting machine-tool. A model of a feed drive using a dc servomotor as actuation device was also integrated into this simulation tool. The simulation data were taken from the actual CNC laser cutting machine which was further used for experimental tests.

The proposed simulation diagram allows the user to test the contouring accuracy of the machine and to take corrective actions to improve them by altering the control parameters of the position controller, by means of simulation. Simulation results had shown significant differences between the reference and the actual circular trajectory. The approach for reducing these differences, and consequently to improve the machining accuracy when processing complex trajectories, was to increase the proportional gain of the position controller on one axis.

Experimental researches were also performed, by cutting a circular a contour on an industrial CNC laser cutting machine and extracting geometrical information by means of a joint methodology of measuring and processing "points cloud" files format.

The experimental results have validated the simulations, showing that the proposed simulation diagrams may be used as accurate tools in the initial stages of a hands-on approach process of improving the machinetool dynamic behaviour and accuracy at the workshop level.

## REFERENCES

[1] C.L. Caristan, Laser Cutting Guide for Manufacturing, Society of Manufacturing Engineers, 2003.
[2] C. Bagger, F.O. Olsen, Pulsed mode laser cutting of sheets for tailored blanks, J. Mater. Process. Technol. 115, pp. 131-135, 2001.
[3] E. Craig, Thermal Cutting, în ASM Handbook, vol. 14: Forming and Forging, 9th Edition, 4th Printing, ASM International, 1996.
[4] J.F. Ready, Industrial applications of lasers, Academy Press, Tokyo, 1998.
[5] S.D. El Wakil, Processes and design for manufacturing, PWS Publishing Company, Boston, 1998.
[6] C. Bagger and F.O. Olsen, Pulsed mode laser cutting of sheets for tailored blanks, J. Mater. Process. Technol,. 115, 2001, pp. 131-135.
[7] B.S. Yilbas, Laser cutting quality assessment and thermal efficiency analysis, J. Mater. Process. Technol., 155-156, 2004, pp. 2106-2115.
[8] R. Aloke, V. Girish, R.F. Scrutton, P.A. Molian, A model for prediction of dimensional tolerances of laser cut holes in mild steel thin plates, Int. J. Mach. Tools Manuf. 37(8), 1997 pp. 1069-1078.
[9] K. Astrom, B.Wittenmark, Computer-Controlled Systems: Theory and Design (3rd ed.), Prentice-Hall, Englewood Cliffs, NJ, 1997.
[10] K, Erkorkmaz, and Y. Altintas, High-speed CNC system design, part III: high-speed tracking and contouring control of feed drives, Int. J. Mach. Tools. Manuf., 41, 2001, pp. 1637-1658.
[11] Y. Koren, Advanced Controllers for Feed Drives, CIRP Annals, Vol. 41, No. 2, 1991, pp. 689-698.
[12] R.L. Hecker, G.M. Flores, Q. Xie and R. Haran, Servocontrol of Machine-Tools: A Review, Latin American Applied Research, 38, 2008, pp. 85-94.
[13] Y.T. Shin, C.S. Chen and A.C. Lee, A novel crosscoupling control design for Bi-axis motion, Int. J. Mach. Tools. Manuf., 42(14), 2002, pp. 1539-1548.
[14] J.H. Chin, Y.M. Cheng and J.H. Lin, Improving contour accuracy by Fuzzy-logic enhanced cross-coupled precompensation method, Robot. Comput. Integr. Manuf, 20(1), 2004, pp. 65-76.
[15] M.Y. Cheng and C.C. Lee, Motion controller design for contour following tasks based on real-time contour error estimation, IEEE Trans. Ind. Electron., 54(3), 2007, pp.1686-1695.
[16] R.E. Breaz, O.C. Bologa, V. Oleksik, and G. Racz, Computer Simulation for the Study of CNC Feed Drives Dynamic Behavior and Accuracy, Proceedings of the IEEE Region 8 EUROCON 2007, International Conference on "Computer as a tool", Warsaw, Poland, September 2007, pp. 2229-2233.
[17] R.E. Breaz, S. Tîrnovean, C. Biriş, O.C. Bologa, Improving the Dynamic Behavior and Working Accuracy of the CNC Laser Cutting Machines, 12th International Conference on Control, Automation, Robotics and Vision, ICARCV 2012, 5-7 December, Guangzhou, 2012
[18] PowerMotion ${ }^{\circledR}$ Products, Servo Products Specification Guides, GFH-001B, GE Fanuc Manual, September 1998.


[^0]:    * Corresponding author: University of Sibiu, Engineering Faculty, Sibiu, 4 Emil Cioran, Sibiu, Romania
    Tel.:0269216062/1455;
    E-mail addresses: radu.breaz@ulbsibiu.ro (R. Beaz),
    cristina.biris@ulbsibiu.ro (C. Biris), octavian.bologa @ulbsibiu.ro (O. Bologa), tarnovean@yahoo.com (S. Tîrnoveanu \}

