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GENERAL ALGORITHM FOR COMPONENT DESIGN-OPTIMIZATION FOR MANUFACTURING EQUIPMENT CONSTRUCTION

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Abstract: *The modernization of manufacturing processes involves by all means the development and evolution of the technological processing, assembly and control equipment by developing new solutions in the field of blank and tool orientation and clamping, training, support, guidance, stiffening and transfer devices.*
Multi-criteria optimization of the technological system is determined by the complexity and dynamic character of its components, laying emphasis on its ability to adapt quickly and efficiently to the variety of the production tasks.
In our research we have developed and used mathematical models and optimization algorithms to design new structures and components for device, equipment and machine-tool construction and modernization.
The systematic analysis of the optimization problems involved in the technological process and equipment is required by the need to highlight the multitude of factors and their interdependence with a view to establishing rational solutions in the field of manufacturing engineering.
The optimization methods presented in this paper, from the optimization of blank surface orientation and clamping, the optimization of orientation and clamping device construction, the optimization of new structure development to the method of automatic generation of the manufacturing equipment structures are important steps in the complex optimization of the technological processes.
As a systematic and objective approach of this paper, we have defined a general graph to describe the structures of the manufacturing equipment and an associated mathematical model used to optimize them.

Key words: *design algorithm, mathematical model, optimization components, processing equipment.*

1. INTRODUCTION

The problem of the technological process optimization, related to the solving method and its complexity requires a thorough analysis and a complete formulation of the problem and of the optimization criterion. The paper [2, 6] emphasizes the fact that the search method of a technological problem optimal solution involves the following steps:

- developing the problem mathematical model;
- defining the optimization criterion;
- designing the solving algorithm;
- solving the problem.

The mathematical modeling of an optimizing problem requires an objective description of the process or the equipment to be solved as well as of the optimization criterion involving the following steps:

- mathematical formulation of the optimization purpose as an analytical expression of a criterion called function-purpose or performance purpose;
- mathematical formulation of the interdependencies between the characteristic elements as analytical relationships representing restrictions or constraints of the real system functioning.

As far as technological processes optimization is concerned, the technological system mathematical modeling as a whole, as well as its components modeling, is not only useful but also more economical because it eliminates many lengthy expensive practical attempts to find the optimal solution.

2. OPTIMIZATION METHODS

The systematic approach of the technological process optimization presented in the papers [2, 7, 5, and 8] is necessary to highlight the multitude of factors in their interdependencies. It is also necessary to emphasize the fact that in industry optimal solutions are not sufficiently developed and the complex optimization of the manufacturing processes and equipment is not at all simple. Most authors in papers published so far [6, 8, 3, 9] have shown significant research on optimizing the blank choice, determining the optimal number and sequence of operations and phases of the technological process, determining optimal tooling allowance, optimizing cutting regime and finding optimal device orientation and clamping schemes for blank processing in machine-tools.

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Our most recent research aims to develop, modernize and increase the efficiency of the manufacturing processes and systems. To this purpose, we have designed and used new computing methods, mathematical models, optimization algorithms, processing methods and original solutions of structures for the construction of processing equipment. All these achievements have resulted in industrial applications which have increased the technological equipment performance and flexibility.

The optimization methods which we have designed and which will be presented in this paper represent local optimizations, being research stages necessary for the technological process and equipment complex optimization.

2.1. Blank orientation and clamping surface optimization

The improvement of manufacturing technologies is conditioned by the development of processing, assembly or control equipment, in which the device is an important component that must ensure the technological system accuracy, productivity and flexibility.

The first optimization stage is shown in Fig. 1 in which, on the basis of the requirements imposed in the work piece drawing and in the plan of operations, SEFA methodology is followed [2, 3] and the optimal orientation and clamping scheme O-OCS (SOF-O) is designed for each operation.

The optimal solution, expressed by means of informational symbols, only takes a rigorous account of all the kinematic, construction, technological and economic aspects in the blank and support element contact area and only a partial account or no account of those connected to the complete structure of the supports.

2.2. Support construction optimization

Informational symbols, necessary for the determination of the optimal orientation and clamping blank surfaces, do not have and cannot have so many suggestive graphic signs for the multitude of support constructive variants used in the design and development of special devices SD (DS) and modular devices MD (DEM). These structures which do not have a correspondent in the

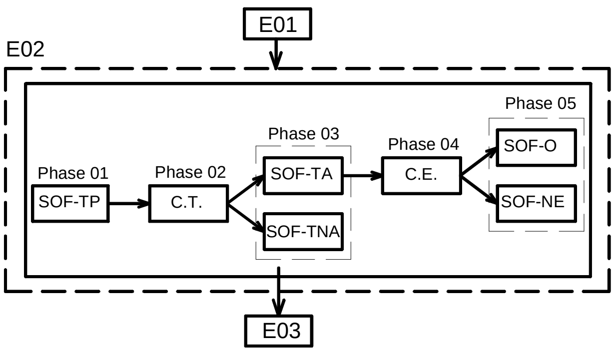


Fig. 1. Optimization phases OCS/SOF.

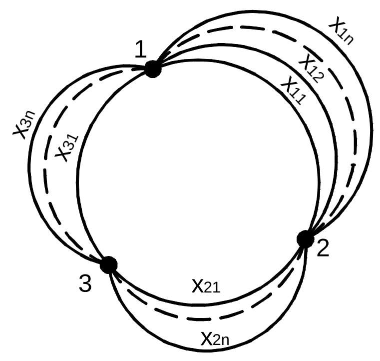


Fig. 2. Technological graph.

informational symbols cannot be analyzed using the SEFA methodology from the point of view of accuracy and characteristics, beyond the blank contact area.

Subsequently, in this paper [5] we have proposed a new method for calculating support construction orientation errors (ε_{ocs}) and a mathematical model for their optimization. The model in its generalized form looks like this:

$$\begin{cases} \min C = \sum_{i=1}^{m}\sum_{j=1}^{n_i} x_{ij}c_{ij} \\ \sum_{j=1}^{n_i} r_{ij}x_{ij} REL\ R_i \\ \sum_{j=1}^{n_i} x_{ij} = 1;\ x_{ij} = 1\ sau\ 0 \\ \dots\dots\dots\dots \end{cases}, \qquad (1)$$

where i = graph node;

m = numbers of nodes;

j = considered variant index;

n_i = number of variants in node i;

REL = relational operator (≤, =, ≥);

x_{ij} = variant j in node i;

r_{ij} = coefficients corresponding to x_{ij} variant for R_i restriction;

R_i = imposed restriction in node i;

C = objective function;

c_{ij} = j variant cost in node i.

This mathematical model must be individualized on the basis of the technological graph in Fig. 2, associated with the construction of a blank orientation and clamping device, in which the three nodes represent the number of symbols for the O-OCS (SOF-O) orientation supports. The constructive variants of the supports in each node will be written with $x_{11}...x_{1n}$ (for node 1), $x_{21}...x_{2n}$ and $x_{31}...x_{3n}$ respectively.

The mathematical model has as a function object the cost of the device (C), and as restrictions accuracy (ε), productivity (p), flexibility (f), the type of manufacturing assimilation (t) and exploitation bahaviour (e). The structure combination of the three types of support which simultaneously fulfill the conditions of imposed restrictions and minimum cost will become the optimal variant.

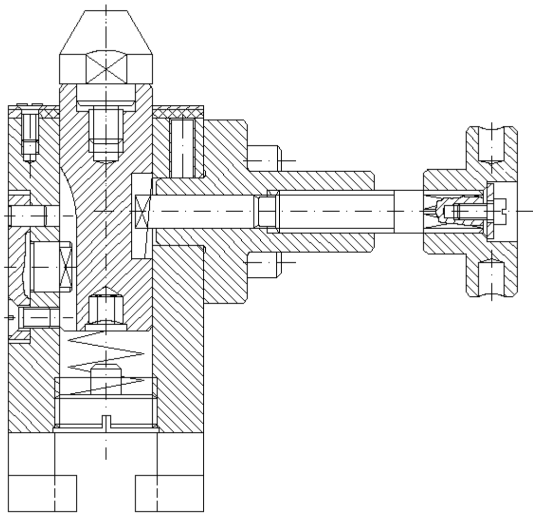


Fig. 3. Multi-function mobile support.

Thus, after establishing O-OCS (SOF-O) by using the SEFA methodology, the stage of structure optimization for the construction of supports and both special SD (DS) and modular devices MD (DEM) is followed.

2.3. New structure development optimization

Modern manufacturing requires the development of new structures of the components used in processing equipment construction.

In the paper [4], the structure evolution can be described using linear complex functions, presented in the specialty literature as transformation functions. Particularized for the considered case, these functions, having the form

$$\zeta(x_1, x_2 ... x_n) \Rightarrow \zeta'(x_1 + \Delta x_1, x_2 + \Delta x_2, ... x_n + \Delta x_n) \quad (2)$$

have allowed the gradual development of components shown on the diagram tree levels as the most important support structures for the construction of special devices SD (DS) and modulation devices MD (DEM).

One of these solutions is the auto-placement and subsequent blocking multi-function mobile support in Fig. 3., in which certain functions, connected with the type of the support element, its positioning and its connection with the system, have been developed with the aim of obtaining new modular structures for the construction of processing devices and equipment.

2.4. Methods of automatic generation of structures for processing equipment construction

Once the optimization stages presented above have been followed, it is necessary for the construction of special and especially modular processing devices and equipment to identify and use structures already existing in the database or to develop structures in the course of designing. They will configure both simple and complex processing systems, automatically and interactively, with a view to determining the optimal solution through visualising the system structure and functioning.

In the paper [1], the analysis of the diversity of support, device and manufacturing equipment variants shows that they are made up of constructive-functional blocks.

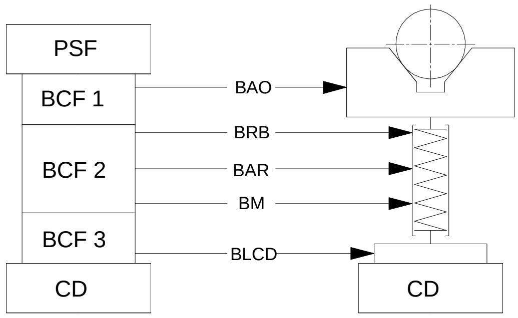


Fig. 4. Constructive-functional model.

We can admit that any technological equipment is generally a reunion of constructive-functional blocks $(CFB)_k$ $((BCF)_k)$, with specific composition laws $(SCL)_k$ $((LCS)_k)$, each block being able to contain sub-blocks. For a support, the general constructive-functional model is the one shown in Fig. 4.

The keeping or the replacement of a constructive-functional block leads to the existence of a multitude of variants of technological equipment subject to the optimization criteria.

To solve the problem by means of informatics, the following steps will be taken:

1. The support drawing (or that of another structure) will be done with AutoCAD design;
2. Elements making up the support will turn into blocks to obtain the intelligent drawing;
3. The multitude of supports which are based on the general constructive-functional model will be automatically generated.

To solve steps 2 and 3, an algorithm written in Visual Basic is used, following the next steps: designing the user interface, setting properties, forms and controls, writing the code and testing the application.

3. GENERAL ALGORITHM FOR COMPONENT DESIGN-OPTIMIZATION FOR PROCESSING EQUIPMENT CONSTRUCTION

The methods presented in Chapter 2 of the present paper are local optimizations which we consider pre-optimization stages of the processing equipment. They represent a gradual approach of complicated problems related to the complex optimization of the manufacturing processes in the car industry.

3.1. The general design-optimization algorithm of the processing equipment

For a more rigorous and more comprehensive analysis of the processing devices and equipment, from simple components to complex manufacturing systems, we propose the design-optimization algorithm shown in Fig. 5.

It is based on O-OCS (SOF-O) system in SEFA methodology as well as on the other optimizing methods we have presented, from which a diversity of special,

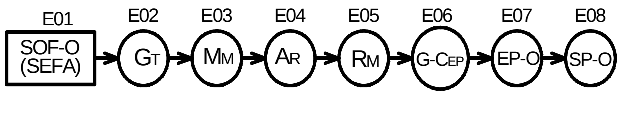


Fig. 5. Design-optimization algorithm.

specialized or modularized structures result, involving the following steps:

E_{01}. Knowing or determining O-OCS (SOF-O), applying SEFA methodology, expressed as information symbolization of the three types of support;

E_{02}. Setting the technological graph of the processing equipment constructive variants;

E_{03}. Formulating the mathematical model on the basis of the graph, mentioning the component elements (the objective function, restrictions, other relations);

E_{04}. Defining the algorithm for solving the mathematical model;

E_{05}. Solving the mathematical model;

E_{06}. Generating and configuring the processing equipment structures;

E_{07}. Establishing the optimal solution of the processing equipment components;

E_{08}. Establishing the optimal solution of complex processing systems.

3.1.1. Setting the technological graph

To describe the structures of the processing equipment, we have shown in Fig. 6 the general technological graph of constructive variants, from the simple variants of the support type to the more complex variants of the processing system type. The nodes of the graph represent the types of structures analyzed and the arcs of the graph are their constructive variants:

Node 1 – variants of support 1

Node 2 – variants of support 2 ,

Node 3 – variants of support 3 ,

Node 4 – variants of the clamping mechanism,

Node 5 – variants of the multi-function mechanism,

Node 6 – variants of the blank orientation and clamping device OCD (DOF),

Node 7 – variants of the transfer device TD (DT),

Node 8 – variants of the complex processing system.

The significance of the arcs of the graph as constructive variants of structures is the following:

Node 1:

x_{11} – special, mobile bolt support, with self-placement and blocking

x_{12} – modularized, mobile bolt support, with self- placement and blocking

x_{13} – special, mobile bolt support in wear bush, with self- placement and blocking,

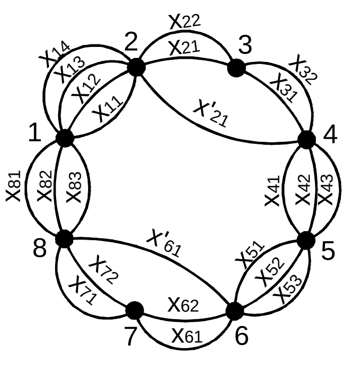


Fig. 6. The primary technological graph.

x_{14} – mobile bolt support in wear bush with self- placement and hidroplast blocking,

Node 2:

x_{21} – flat base support with support plugs ,

x_{22} – flat base support with support plates .

Node 3:

x_{31} – self-centering mechanism ,

Node 4:

x_{41} – screw clamping mechanism,

x_{42} – screw and lever clamping mechanism,

x_{43} – hidroplast clamping mechanism,

Node 6:

x_{61} – special blank orientation and clamping device with motherboard and moulded monobloc body

x_{62} – special blank orientation and clamping device made up of modular elements (DEM)

Node 8:

x_{81} – special milling-centering machine

x_{82} – milling-centering aggregate machine

x_{83} – modular milling-centering machine

The conceptual development of new components of the processing equipment presented in Subchapter 2.3, makes it possible to develop multifunction structures whose characteristics and performance are better than those of the existing structures. The constructive variant x_2' is a multifunctional component of the orientation and clamping device OCD (DOF) which include a self-centering support and a flat base. Thus, in the secondary technological graph, x_{21}' is the rational solution as an alternative to the node 2 and node 3 combinations. Likewise, arch x_{61}' represents a construction variant of processing equipment which, in the same solution, brings together the blank orientation and clamping device OCD (DOF) and the transfer device TD (DT), attached to the basic components of the machine-tool MT (MU).

The general technological graph of the processing equipment construction variants has certain characteristics:

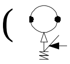
- in the graph presented above we have shown structures of orientation supports, clamping mechanism, multifunctional components, orientation and clamping devices, transfer devices and basic components of machine-tool; this graph can be extended, adding nodes for other structures (components for tool guidance, tools, tool training devices) or it can be simplified reducing the analysis to a small number of nodes and types of structures;
- the general technological graph is defined from the point of view of the design-optimization logic and order from simple to complex, so that the optimization of a processing complex system includes the optimization of the other components of every node;
- the following of the general technological graph represents a systematic approach of the optimization problems of the processing equipment, but, at the same time, allows a local graph to be described in every node in which specific structures will be ana-

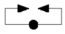
lyzed and developed using the optimization methods presented in Chapter 2;

- in every node of the graph only the specific elements to every type of structure of the respective node are analysed, without making any reference to those taken from the previous nodes which have already been evaluated;
- the construction variants of the analysed structures can be special, specialized or modularized or their combinations for simple components as well as for complex systems.

3.1.2. Formulating the mathematical model

To formulate the mathematical model, the general technological graph in Fig. 6, also named primary graph must be converted into a secondary graph, all the technological routes representing Hamiltonian roads [6], as in Fig. 7, where:

Node 1 –variants of support 1 ()

Node 2 – variants of multifunctional support 2 (+ ),
Node 3 – variants of the clamping mechanism,
Node 4 – variants of the multifunctional mechanism,
Node 5 – variants of the orientation and clamping device OCD (DOF) integrated to the transfer device DT (DT),
Node 6 – variants of the complex processing system.

In this secondary graph the new variables x'_{22}, $x'_{23}...x'_{26}$ codify groups of structures in the following way:

$$\begin{cases} x'_{22} = x_{21} + x_{31} \\ x'_{23} = x_{21} + x_{32} \\ x'_{24} = x_{22} + x_{31} \\ x'_{25} = x_{22} + x_{32} \end{cases}, \qquad (3)$$

where the signs (=) and (+) do not have, in all cases, a strict mathematical interpretation (if $x'_{22} = 1$, then $x_{21} = 1$ and $x_{31} = 1$; if $x'_{23} = 0$, then $x_{21} = 0$ and $x_{32} = 0$).

The other codification of the structures of node 5 and 6 has the same significance.

The mathematical model of the general technological graph associated with the processing equipment constructive variants, in which we propose cost (*C*) as the objective function and accuracy (ε), productivity (*P*), flexibility (*F*), fabrication preparation time (*T*) and exploitation behaviour (*E*) as restrictions, is the following:

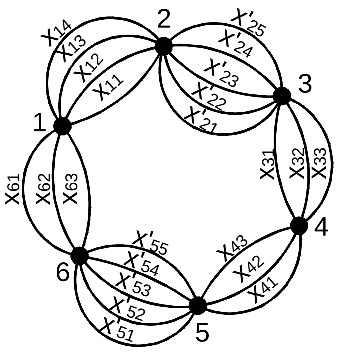


Fig. 7. Secondary technological graph.

$$\begin{cases} \min C = c_{11}x_{11} + c_{12}x_{12} + ... + c'_{51}x'_{51} + ... + c_{63}x_{63} \\ \varepsilon_{11}x_{11} + \varepsilon_{12}x_{12} + \varepsilon_{13}x_{13} + \varepsilon_{14}x_{14} \le \varepsilon_{1\max} \\ p_{11}x_{11} + p_{12}x_{12} + p_{13}x_{13} + p_{14}x_{14} \ge p_{1\min} \\ f_{11}x_{11} + f_{12}x_{12} + f_{13}x_{13} + f_{14}x_{14} \ge f_{1\min} \\ t_{11}x_{11} + t_{12}x_{12} + t_{13}x_{13} + t_{14}x_{14} \le t_{1\max} \\ e_{11}x_{11} + e_{12}x_{12} + e_{13}x_{13} + e_{14}x_{14} \ge e_{1\min} \\ \sum_{j=1}^{4} x_{1j} = 1; \;\; x_{1j} = 1 \;\; sau \;\; 0; \;\; j \in \{1,2,3,4\} \\ .... \\ \varepsilon_{41}x_{41} + \varepsilon_{42}x_{42} + \varepsilon_{43}x_{43} + \varepsilon_{44}x_{44} \le \varepsilon_{4\max} \\ .... \\ f'_{51}x'_{51} + f'_{52}x'_{52} + f'_{53}x'_{53} + f'_{54}x'_{54} \ge f_{5\min} \\ .... \end{cases} \qquad (4)$$

After interpreting the x_j variables, the next problem is that of mathematical programming in integer numbers, of the binary decisions. For this reason, we use the mathematical programming method in bivalent variable for solving.

3.1.3. Algorithm for solving the mathematical model

The algorithm for solving the system involves the following steps:

P01. The initial data (constant and variable) which will be introduced into the system are given on the basis of the mathematical model;

P02. For every node of the graph, the variants of the vectors which fulfill the condition $\sum_{j=1} x_{ij} = 1$; $x_{ij} = 1$ or 0 are identified;

P03. The model restrictions are checked for every vector, keeping those which fulfill the imposed conditions;

P04. The function object is calculated for every possible vector combination of nodes which verifies the restrictions;

P05. The smallest function value and the vector combination leading to it are chosen.

In case there is a relatively small number of constructive variants and restrictions imposed on the system, this algorithm can be carried out manually, only for more complicated cases is the computer used.

In this case, the computer needs that the mathematical model and the technological graph configuration should be written in matrix form:

$$\begin{cases} A^{+}X = U \\ A^{+}(\varepsilon X) \le \varepsilon_{\max} \\ A^{+}(PX) \ge P_{\min} \\ A^{+}(FX) \ge F_{\min} \\ A^{+}(TX) \le T_{\max} \\ A^{+}(EX) \ge E_{\min} \\ \min(CX) \end{cases}, \qquad (5)$$

where, $A = \{a_{ij}\}$ is the incidence matrix associated with the graph in which a_{ij} elements are established in the following way:

$$a_{ij} = \begin{cases} +1 & \text{if point } i \text{ is the initial extremity of arc } x_{ij}, \\ -1 & \text{if point } i \text{ is the final extremity of the arc,} \\ 0 & \text{if point } i \text{ is an extremity of the arc.} \end{cases}$$

A^+ = the positive side of the incidence matrix,
X = column vector of the constructive variants

$$X = [x_{11}, x_{12}, x_{13}, ..., x_{33}, ... x_{63}]^t$$

U = column matrix with m elements equal to 1,

$$U = [1, 1, 1, ..., 1]^t$$

To express the other restrictions, the square matrices of the form

$$\begin{pmatrix} \varepsilon_{11} & 0 & 0 & 0 & 0 & ... & ... & 0 \\ 0 & \varepsilon_{12} & 0 & 0 & 0 & ... & ... & 0 \\ 0 & 0 & \varepsilon_{13} & 0 & 0 & ... & ... & 0 \\ 0 & 0 & 0 & \varepsilon_{14} & 0 & ... & ... & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{15} & ... & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & 0 \\ 0 & 0 & 0 & 0 & 0 & ... & \varepsilon_{62} & 0 \\ 0 & 0 & 0 & 0 & 0 & ... & 0 & \varepsilon_{63} \end{pmatrix} \qquad (6)$$

are introduced.

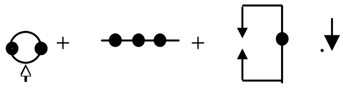
Finally, after solving the system of inequations (5), the optimal construction solution of the processing equipment is obtained. In this optimal solution, all the equipment components, from simple supports to complex systems, are setting by means of the gradual optimization on the basis of the technological graph.

Our initial research, as a work method, has been extended also to the customizations needed to optimize fitted with devices of an entire technological process.

4. CASE STUDY

In order to achieve the general algorithm for design and optimization of machining equipment, we present a case study, in which we analyse only a sequence of secondary technology graph where variants of structures of nodes 1, 2 and 3 define the construction of a device.

In Fig. 8, the workpiece of lever type for direction system of an automobile needs a guidance device for the drilling operation Ø10 $^{\pm 0.075}$ whose optimal guidance and fixing scheme, established with the SEFA, is:

SOF-O: + 

Form the optimum orientation and fixing scheme (SOF-O) one develops design optimization algorithm of construction of the device (or of any other equipment which includes the device).

It is considered a small number of constructive variants of structures and limitations (accuracy ε and flexibility f), but sufficiently suggestive and covering for the confirmation of the theoretical principle and mathematical calculus.

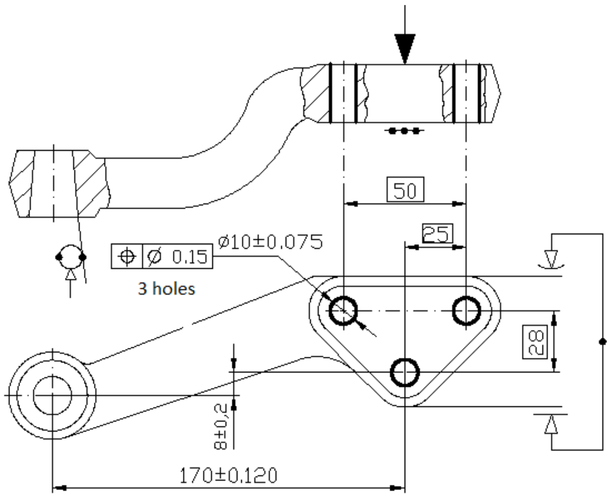
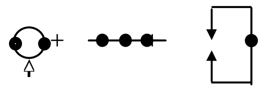
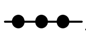


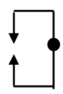
Fig. 8. Construction optimization of the orientation supports of the device.

Variants of construction of structures supports for the set SOF-O are coded as follows:

(


x_3 – bolt in steps with short guidance on d_1 and d_2 (double guidance on different diameters);
x_4 – lis bolt directly in the device body;
x_6 – lis bolt in wear bushing with adjustable backlash or locking by hidroplast;


x_8 – surface materialized through seating caps (or pads);


x_{10} – self-centering systems with left-right screw and jaws
x_{11} – self-centering systems with left-right screw and jaws with adjustable pad.

One determines the graph of constructive variants (Fig. 9) and associated mathematical model, taking into account as restrictions only processing accuracy (ε) and flexibility (f):

$$\begin{cases} \min C = C_3 x_3 + C_4 x_4 + C_6 x_6 + C_8 x_8 + C_{10} x_{10} \\ \quad + C_{11} x_{11} \\ \varepsilon_3 x_3 + \varepsilon_4 x_4 + \varepsilon_6 x_6 \le \varepsilon_{\max 1} \\ f_3 x_3 + f_4 x_4 + f_6 x_6 \ge 1 \\ \sum_i x_i = 1, \quad x_i = 1 \vee 0, \quad i \in \{3, 4, 6\} \\ \varepsilon_8 x_8 \le \varepsilon_{\max 2} \\ f_8 x_8 \ge 1 \\ \sum_i x_i = 1, \quad x_i = 1 \vee 0 \\ \varepsilon_{10} x_{10} + \varepsilon_{11} x_{11} \le \varepsilon_{\max 3} \\ f_{10} x_{10} + f_{11} x_{11} \ge 1 \\ \sum_i x_i = 1, \quad x_i = 1 \vee 0, \quad i \in \{10, 11\} \end{cases}$$

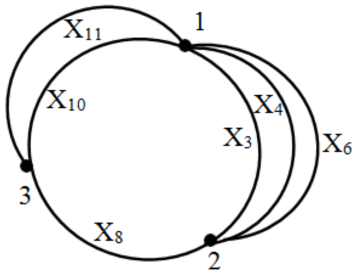


Fig. 9. Graph of constructive varians.

Setting the coefficients:

- for ε_3 the relation:

$\varepsilon_{ocs}^{max} = 2(\sqrt{e_1^2 + e^2 + e_2^2} + jf_{p/2} + HK)$, from work [5] is used, where: $e_1 = 0.01$; $e = 0.01$; $e_2 = 0.01$;

$jf_p = 0.724 \cdot 0.029 = 0.0209$, for the case of $\phi 16 \frac{H_7}{h_6}$

with: $l_c = 1.5\,d$ and $H_c = 19$, $HK = 0.016617$

$$\varepsilon_{ocs(3)}^{max} = 2(\sqrt{0.01^2 + 0.01^2 + 0.01^2} + \frac{0.0209}{2} + 0.0166) = 0.0888;$$

$\varepsilon_3 = 0.0888$;

- for ε_4, from the same relation, customizing $e = 0$, $e_2 = 0$, relation is:

$\varepsilon_{ocs(4)}^{max} = 2(\sqrt{0.01^2} + \frac{0.0209}{2} + 0.0166) = 0.074$; $\varepsilon_4 = 0.074$;

- for ε_6, from the same relation, with customizing: $e_2 = 0$, $jf_p = 0$, $HK = 0$,

$\varepsilon_{ocs(6)}^{max} = 2\left(\sqrt{0.01^2 + 0.01^2} + 0 + 0\right) = 0.0282$; $\varepsilon_6 = 0.0282$;

- for ε_{max1} it is used the relation:

$$\varepsilon_{max\,1} = \sqrt{\left(\frac{1}{3}T_{p1}\right)^2 - \left(\varepsilon_o^2 + \varepsilon_f^2\right)},$$ from work [5],

where: $T_{p1} = 0.24$ (for $170^{\pm 0,120}$), $\varepsilon_o = 0$, $\varepsilon_f = 0.01$.

Thus,

$$\varepsilon_{max\,1} = \sqrt{\left(\frac{1}{3} \cdot 0.24\right)^2 - \left(0^2 + 0.01^2\right)} = $$
$$= \sqrt{0.08^2 - 0.01^2} = \sqrt{0.0063} = 0.079,$$

$\varepsilon_8 = 0.015$ (represents deviation of parallelism of the sitting surface);

$\varepsilon_{max\,2} = 0.075$ (the inclination of the axis of the hole as a displacement within the limits of its tolerance field $\phi 10^{\pm 0,075}$).

$$\varepsilon_{10} = \left(\frac{0.027}{2} + \frac{0.015}{2}\right) + 0.03 + 0.01 = 0.061,$$

where: $\varepsilon_{10} = 0.061$ and: $\frac{0.027}{2} + \frac{0.015}{2} =$ key play;

0.03 = non perpendicularity of the jaw;

0.01 = dimension tolerance of the collar for the contact element with the part (caps);

$$\varepsilon_{11} = \left(\frac{0.027}{2} + \frac{0.015}{2}\right) = 0.021, \varepsilon_{11} = 0.021;$$

$$\varepsilon_{max\,3} = \sqrt{\left(\frac{1}{3}T_{p3}\right)^2 - \left(\varepsilon_o^2 + \varepsilon_f^2\right)},$$

where $T_{p3} = 0.4$ (for the dimension $8^{\pm 0.2}$).

Determination of coefficients of flexibility is made by comparison of variants taking into account the capacity of adaptation of the production variation.

It is considered for a support that cannot be reused $f = 0$;

$f_3 = 1$, because for this type of support another bolt because with different diameter cannot be adapted because the centering in the hole (body) cannot be modified.

$f_4 = 3$, because it can accommodate multiple mobile items with different diameters outside the hole

$f_6 = 5$, because they can adapt more hole diameters in outside of the hole and can be used in more precision conditions due to the play compensation;

$f_8 = 2$, because the setting caps adapt more easily than pads;

$f_{10} = 4$, because it is self-centering with adjustment in wide limits but cannot compensate for the errors of the contact elements as deviations from the head height;

$f_{11} = 5$, it has all the benefits of version 10 but, in addition, the adjustment of the contact elements with the part.

It is accepted $f = 1$ because theoretically and practically it is possible to appear a semifinish with the same dimension.

Using these coefficients the system of inequations is rewritten and solved.

$$\begin{cases} \min C = C_3 x_3 + C_4 x_4 + C_6 x_6 + C_8 x_8 + C_{10} x_{10} \\ \quad + C_{11} x_{11} \\ 0.088\, x_3 + 0.074\, x_4 + 0.0282\, x_6 \le 0.079 \\ x_3 + 3\, x_4 + 5\, x_6 \ge 1 \\ x_3 + x_4 + x_6 = 1, \quad x_3, x_4, x_6 = 1 \vee 0 \\ 0.015\, x_8 \le 0.075 \\ 2\, x_8 \ge 1 \\ x_8 = 1 \\ 0.061\, x_{10} + 0.021\, x_{11} \le 0.133 \\ 4\, x_{10} + 5\, x_{11} \ge 1 \\ x_{10} + x_{11} = 1, \quad x_{10}, x_{11} = 1 \vee 0 \end{cases}$$

4.1. Algorithm for matricial solving of the technological graph for the case study

Starting from the technological graph in Fig. 9, we build the mathematical model. The incidence matrix associated to the graph arcs denoted by A is :

Arcs (columns) Nodes (rows)

$$A = \begin{array}{c} \begin{array}{cccccc} x_3 & x_4 & x_6 & x_8 & x_{10} & x_{11} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 0 & -1 & -1 \\ -1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \qquad (7)$$

The graph is browsed clockwise.

The positive part of the incidence matrix is written (A^+), which takes in consideration, in browsing the graph clockwise, only the elements that starts from nodes denoted by +1; the negative elements (−1) in this case they are becoming 0:

$$\begin{array}{cc} & \begin{matrix} x_3 & x_4 & x_6 & x_8 & x_{10} & x_{11} \end{matrix} \\ A^+ = & \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array} \quad (8)$$

To solve the questioned optimization problem and shown in the technological graph in Fig. 9, the mathematical relationships that exist between the analyzed structures as variants of the device's construction and optimization criteria selected as objective and function restrictions are written in matrix form.

It is considered that the sum of arcs of the construction variants within a node is 1:

$$(A^+)(X) = (U)\,, \quad (9)$$

where A^+ is the positive part of the incidence matrix,

X – column vector of the constructive variants;

U – column vector that has elements equal to 1 ($U = (1 \;\; 1 \;\; 1)^T$) for the number of the nodes of the technological graph:

For the considered case, one writes:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_3 \\ x_4 \\ x_6 \\ x_8 \\ x_{10} \\ x_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (10)$$

and gets after calculation:

$$\begin{pmatrix} x_3 + x_4 + x_6 \\ x_8 \\ x_{10} + x_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad (11)$$

whence the system which establishes the relationship between construction variants of the device:

$$\begin{cases} x_3 + x_4 + x_6 = 1 \\ x_8 = 1 \\ x_{10} + x_{11} = 1 \end{cases}. \quad (12)$$

The analyzed structures coded by $x_3, x_4, \ldots x_{10}, x_{11}$ are bivalent variables taking values 1 or 0. The structure matrix for equations (12) is built putting the bivalence condition:

$$Mb = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}. \quad (13)$$

For analysis of device variants derived from combining structures $(3*1*2 = 6)$ one writes the relation:

$$(Mb)(X) = (V) \quad (14)$$

where V is the column vector of the device variants:

After calculations the system of the construction variants of the device is obtained:

$$\begin{cases} x_3 + x_8 + x_{10} = V_1 \\ x_3 + x_8 + x_{11} = V_2 \\ x_4 + x_8 + x_{10} = V_3 \\ x_4 + x_8 + x_{11} = V_4 \\ x_6 + x_8 + x_{10} = V_5 \\ x_6 + x_8 + x_{11} = V_6 \end{cases}. \quad (15)$$

To express restrictions imposed to the system, the relations for the two chosen selection criteria: accuracy guidance expressed as actual guidance error ε and the flexibility of adaptation (*f*) of the equipment. We chose only two restrictions for ease of calculation.

I. Precision constraint, in matrix form, is:

$$(A^+)(\varepsilon)(X) \le (\varepsilon_{\max})\,, \quad (16)$$

where $\varepsilon_{\max}$ is the error limit of precise condition:

The coefficients

$$\varepsilon_3, \varepsilon_4 \ldots \varepsilon_{10}, \varepsilon_{11}, \varepsilon_{1\max}, \varepsilon_{2\max}, \varepsilon_{3\max}$$

are calculated and introduced in relation:

$\varepsilon_3 = 0.088$, $\varepsilon_4 = 0.074$, $\varepsilon_6 = 0.0282$, $\varepsilon_8 = 0.015$,

$\varepsilon_{10} = 0.061$, $\varepsilon_{11} = 0.021$, $\varepsilon_{1\max} = 0.079$,

$\varepsilon_{2\max} = 0.075$, $\varepsilon_{3\max} = 0.133$.

We get:

$$\begin{pmatrix} 0.088x_3 + 0.074x_4 + 0.0282x_6 \\ 0.015x_8 \\ 0.061x_{10} + 0.021x_{11} \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix}. \quad (17)$$

The following system of inequations is obtained:

$$\begin{cases} 0.088x_3 + 0.074x_4 + 0.0282x_6 \le 0.079 \\ 0.015x_8 \le 0.075 \\ 0.061x_{10} + 0.021x_{11} \le 0.133 \end{cases}. \quad (18)$$

4.1.1. Checking the restrictions for the structures that comprise the device options and meet the requirement of bivalence.

I. The precision condition has the form (16).

Device variant V_1:

The following calculation shall be carried out:

First row of the bivalence matrix

$$\begin{pmatrix} 0.088 & 0.074 & 0.0282 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.015 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.061 & 0.021 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0.088 \\ 0.015 \\ 0.061 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} False \\ True \\ True \end{pmatrix}. \qquad (19)$$

Conclusion: For variant V_1 not all structures satisfy the precision condition.

All variants results for restriction of precision are presented in Table 1.

II. The restriction of flexibility in matrix form is:

$$(A^{+})(F)(X) \ge (F_{\max}), \qquad (20)$$

where $F_{\max}$ is the limit error of the flexibility conditions:

$$\begin{pmatrix} f_3 x_3 + f_4 x_4 + f_6 x_6 \\ f_8 x_8 \\ f_{10} x_{10} + f_{11} x_{11} \end{pmatrix} \ge \begin{pmatrix} f_{1\max} \\ f_{2\max} \\ f_{3\max} \end{pmatrix} \qquad (21)$$

The coefficients $f_3, f_4 ... f_{10}, f_{11}, f_{1\max}, f_{2\max}, f_{3\max}$ are calculated and introduced in relation:

$f_3 = 1$, $f_4 = 3$, $f_6 = 5$, $f_8 = 2$, $f_{10} = 4$, $f_{11} = 5$

$f_{1\max} = 1$, $f_{2\max} = 1$, $f_{3\max} = 1$,

$$\begin{pmatrix} x_3 + 3x_4 + 5x_6 \\ 2x_8 \\ 4x_{10} + 5x_{11} \end{pmatrix} \ge \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} x_3 + 3x_4 + 5x_6 \ge 1 \\ 2x_8 \ge 1 \\ 4x_{10} + 5x_{11} \ge 1 \end{cases}. \qquad (22)$$

4.1.2. Checking the restrictions for the structures that comprise the device options and meet the requirement of bivalence.

The flexibility condition has the form:

$$(A^{+})(F)(X) \ge (F_{\max}) \qquad (23)$$

Device variant V_1: First row of the bivalence matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \ge \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad (24)$$

The calculation shall be carried out:

$$\begin{pmatrix} 1 & 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \ge \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \ge \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} True \\ True \\ True \end{pmatrix} \qquad (25)$$

Conclusion: all structures satisfy the precision condition.

All variants results for restriction of flexibility are presented in Table 2.

From the analysis of precision and flexibility restrictions the system of inequations is satisfied by the following types of structures, which give the following combinations of device as follows:

$$\begin{aligned} V_3 &= x_4 + x_8 + x_{10}; V_4 = x_4 + x_8 + x_{11}; \\ V_5 &= x_6 + x_8 + x_{10}; V_6 = x_6 + x_8 + x_{11} \end{aligned}. \qquad (26)$$

III. Of the solutions in the relationship (55) the one that minimizes the cost of objective function (the end function or performance) is chosen. The expression of the costs is put in matrix-form:

$$[Mb]\,[C] = [Cv], \qquad (27)$$

where:

Mb – is the bivalence matrix of the structure types that satisfy the restriction s of precision and flexibility (those from V_3, V_4, V_5 and V_6);

C – column matrix of the costs for structure variants;

Cv – column matrix of the costs for variants of the analyzed device variants.

We obtain:

Table 1

Precision condition

Device variant V_1:	**Device variant V_2**	**Device variant V_3**
$\begin{pmatrix} 0.088 \\ 0.015 \\ 0.061 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} False \\ True \\ True \end{pmatrix}$ For variant V_1 not all structures satisfy the precision condition.	$\begin{pmatrix} 0.088 \\ 0.015 \\ 0.061 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} False \\ True \\ True \end{pmatrix}$ For variant V_3 all structures satisfy the precision condition.	$\begin{pmatrix} 0.074 \\ 0.015 \\ 0.061 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} True \\ True \\ True \end{pmatrix}$ For variant V_3 all structures satisfy the precision condition.
Device variant V_4	**Device variant V_5**	**Device variant V_6**
$\begin{pmatrix} 0.074 \\ 0.015 \\ 0.021 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} True \\ True \\ True \end{pmatrix}$ For variant V_4 all structures satisfy the precision condition.	$\begin{pmatrix} 0.0282 \\ 0.015 \\ 0.061 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} True \\ True \\ True \end{pmatrix}$ For variant V_5 all structures satisfy the precision condition.	$\begin{pmatrix} 0.0282 \\ 0.015 \\ 0.021 \end{pmatrix} \le \begin{pmatrix} 0.079 \\ 0.075 \\ 0.133 \end{pmatrix} \Rightarrow \begin{pmatrix} True \\ True \\ True \end{pmatrix}$ For variant V_6 all structures satisfy the precision condition.

Table 2

Flexibility condition

Device variant V_1:	Device variant V_2	Device variant V_3
$\begin{pmatrix}1\\2\\4\end{pmatrix} \ge \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}True\\True\\True\end{pmatrix}$	$\begin{pmatrix}1\\2\\5\end{pmatrix} \ge \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}True\\True\\True\end{pmatrix}$	$\begin{pmatrix}3\\2\\4\end{pmatrix} \ge \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}True\\True\\True\end{pmatrix}$
Device variant V_4	**Device variant V_5**	**Device variant V_6**
$\begin{pmatrix}3\\2\\5\end{pmatrix} \ge \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}True\\True\\True\end{pmatrix}$	$\begin{pmatrix}5\\2\\4\end{pmatrix} \ge \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}True\\True\\True\end{pmatrix}$	$\begin{pmatrix}5\\2\\5\end{pmatrix} \ge \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \begin{pmatrix}True\\True\\True\end{pmatrix}$

All structures satisfy the precision condition.

$$\begin{pmatrix}0&1&0&1&1&0\\0&1&0&1&0&1\\0&0&1&1&1&0\\0&0&1&1&0&1\end{pmatrix}\begin{pmatrix}C_3\\C_4\\C_6\\C_8\\C_{10}\\C_{11}\end{pmatrix}=\begin{pmatrix}Cv_3\\Cv_4\\Cv_5\\Cv_6\end{pmatrix} \qquad (28)$$

Terms $C_3, C_4, C_6, C_8, C_{10}, C_{11}$ of the objective function represent the costs in RON of construction and operation of the device supports of the composition of the four analysed variants Cv_3, Cv_4, Cv_5, Cv_6. The values calculated from literature are: C_3 = 116.92; C_4 = 105.09; C_6 = 128.75; C_8 = 22.28; C_{10} = 238.43; C_{11} = 250.71.

By replacing them in relation (28) we obtain

$$\begin{pmatrix}105.09+22.28+238.43\\105.09+22,28+250.71\\128.75+22.28+238.43\\128.75+22.28+250.71\end{pmatrix}=\begin{pmatrix}365.8\\378.08\\389.46\\401.74\end{pmatrix}=\begin{pmatrix}Cv_3\\Cv_4\\Cv_5\\Cv_6\end{pmatrix}. \qquad (29)$$

The condition of minimum of the cost leads to the value C_{V3} for the optimum device variant V_3 whose components is defined by the structures:

$$V_{3optim} = (x_4, x_8, x_{10}). \qquad (30)$$

5. CONCLUSIONS

Product quality, work productivity, machine-tools loading degree, technological process mechanization and automation, new products in the manufacturing assimilation and making production cost-effective are mostly determined by the technological equipment design, construction and exploitation.

Our latest research aims to develop new structures whose characteristics and performance are better than those of the existing ones, for the construction of the technological system components. They are based on the optimization methods described in Chapter 2 which condition and complete each other.

The authors' original contribution and the novelty of this paper is connected to the formulation of a general algorithm for processing equipment design-optimization, the formulation of a general technological graph and of the associated mathematical model. It is an overview, a systematic approach, which permits the local optimization of every component, from simple to complex, as well as of the whole technological system, intended for the performance of an operation or of a group of technological operations aimed to perform an operation or a group of technological operations.

Optimization has been done for a limited number of nodes (the three types of supports of the device) and regarding only two restrictions (precision and flexibility) for the amount of calculation not to too large; as we stated in the paper, the optimization can be done in each node of the graph or for entire graph.

This paper will be followed by another dealing with the presentation and choice of types of structures for the construction of devices and equipment for processing, but also by a third paper in which we intend to propose a computer program for solving the inequation system regardless of its complexity.

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