OPTIMAL SERVOMOTOR SELECTION ALGORITHM FOR INDUSTRIAL ROBOTS AND MACHINE TOOLS NC AXIS

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Abstract: This paper presents a complete calculus algorithm for selecting the optimal servomotors of the kinematic chains included in the numerically controlled axes (NCA) of machine tools (NCMT) and industrial robots (IR). The algorithm can be applied for both type controlled axes, i.e. industrial robots, and machine tools, as well as either rotation axis or translation axis, regardless of their mechanical structure. The presented calculus algorithm includes six major steps: defining the working cycle motion diagram for the operated mechanical system and complementary input data; identifying the static and dynamic forces applied to the driven system during operation and the specific mechanical structure on NCA (i.e. specific transfer ratio of each included mechanism); preliminary selection of the driving servomotor by checking the kinematic criterion; determining of the total resistant equivalent load applied on the driven element; secondary selection of the driving servomotor by checking the performance parameters related to full kinematic chain driving (acceleration time, braking time, servomotor's thermal behavior). The motor finally chosen represents the optimum solution for driving the NCA in terms of satisfying all imposed criteria (kinematic criterion, static criterion, dynamic criterion) and complying as well the necessary performance requirements for NCA's electro-mechanic driving system.

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Key words: optimal selection, algorithm, servomotor, industrial robot, machine tools, NC axis.

1. INTRODUCTION

Electric motors sizing, has been largely investigated by many authors especially the last two decades especially due to improvement in servomotors design and performances. From this point of view a impressive number of papers may be cited are references in servomotors sizing process as well as software product may be used for preliminary selection and partially checking of brush / brushless DC and synchronous AC servomotors. However, many of these papers deals only about some specific application's servomotor sizing (for mechatronic systems, IR or NCMT specific NCA or are offering an incomplete or limited perspective for final servomotor's selection / final checking due to their restrictive approach.

That is why starting up from earlier of previously two decades the authors have been involved in investigating the related available calculus methodologies and testing the results that may be obtain from different servomotor's sizing software packages. As result of these works a final calculus algorithm for complete sizing and complex checking of brush / brushless DC and synchronous AC servomotors has been accomplish and included in special dedicated university courses [1].

As concern the most important references from the technical literature presenting the methodology for sizing servomotors one of the most complete methodology is presented in [2] (however, with an special emphasis on motor sizing based only on the total inertia of the mechanical components included in NCA's associated kinematic chain structure, as well as the motor selection process based on the requirements for the nominal and peak torques necessary to be supplied by the servomotor). According to this paper [2] the following motor sizing objectives may be considered: to identify the servomotor able to supply the necessary maximum speed required to achieve the maximum speed of the driven element; to identify the optimal transfer ratio between the motor inertia and total inertia of the driven inertial load; to match, as much as possible, the servomotor's nominal and peak torques with resistant torques generated by the driven load; to achieve the best servomotor performance at the best available price.

Generally [1, 2] from above point of view, the sizing and selection of a servomotor is based on the calculation of torques and inertia required by both the mechanical structure and the kinematics (speed and acceleration conditions) necessary for a specific applications. The motor selected must safely operates the driven element / mechanical structure, while providing sufficient torque and speed. Once the prerequisites needs are established (in terms of specifying NCA's kinematic and dynamics), it is relatively easy to analyze and then to choose the suitable servomotor for a specific application, either

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having at disposal the torque-speed diagram, or the motor specifications. However, some final checking procedures for identifying the optimum selected servomotors are not presented in [2], especially the ones related to selection of the driving servomotor by checking the performance parameters related to full kinematic chain driving (acceleration time, breaking time, as well as servomotor's thermal behavior checking).

For this last purpose servomotor's (thermal behavior checking), it may be found a good approach in paper [3] dealing especially about this aspect. However, even the paper intends to present an full optimum selection calculus algorithm for servomotor selection some aspects related to servomotor's thermal behavior evaluation are questionable (especially for final temperature of the servomotor evaluation) and different NCA structures analysis, than the studied one, are missing as well. Useful applicative aspects from the paper may be remarked in terms of identifying the thermal energy dissipated in DC brush motor windings as well as identifying of the optimal transfer ratio for the gearing system included in NCA's mechanical structure from the point of view of minimizing the thermal energy dissipated in servomotor windings.

As concern the last issues highlighted as insufficient founded in previously mentioned papers a complementary approach on specific issues dealing about detailed servomotor's thermal behavior analyses may be found in [4 and 5]. The servomotor finally chosen following up methodologies presented in these papers represents the optimum solution for an optimum selection of NCA's electric driving servomotors in terms of satisfying imposed criteria for servomotor's performances and as well complying the necessary complete servomotor's thermal behavior evaluation.

Besides all aspects previously presented a complete methodology for optimum selection NCA's electric driving system need to take account about specific NCA's mechanical structure. From this point of view IR's or NCMT's NCA may be classified as translation axes (Figs. 1 and 2) and rotation axes (Figs. 3 and 4).

In structural terms, IR's and NCMT's translation NCAs can be differentiated by the integrated motion's nature transformation mechanism, as follows:

- NCAs provided with ball screw mechanism (Fig. 1);

- NCAs provided with a rack-pinion mechanism (Fig. 2).



Fig. 1. Example 1 - Translation axis including ball screw mechanism: 1 - servomotor; 2 - first coupling 1; 3 - first shaft; 4 - driving gear (z1); 5 - driven gear (z2); 6 - second shaft; 7 - second coupling; 8 -ball screw - bearings partial assembly; 9 - carriage; 10 - object

From a functional perspective, the second category of translation NCAs may be used to achieve any of the stroke lengths besides the first one category of translation NCAs is usually used to achieve a maximum 3 meters length of mobile element's stroke.



Fig. 2. Example 2- Translation axis including rack and pinion mechanism: 1 – servomotor; 2 – first coupling; 3 – first shaft;
4 – driving gear (z1); 5 – driven gear (z2); 6 – second shaft; 7 – second coupling 2; 8 – rack and pinion mechanism;

9 - carriage; 10 - object.



Fig. 3. Example 3. Rotation axis including external gear driving system: 1 - servomotor; 2 - first coupling; 3 - first shaft; 4 - first driving gear (z₁); 5 - first driven gear (z₂); 6 - second shaft; 7 - second driving gear (z₃); 8 - second driven gear (z₄); 9 - third shaft; 10 - second coupling 2; 11 - fourth shaft; 12 - third driving gear (z₅); 13 - third driven gear (z₆); 14 - rotary

table; 15 – object.



Fig. 4. Example 4. Rotation axis including internal gear driving system: 1– servomotor; 2 – first coupling; 3 – first shaft;4 –first driving gear (z_1); 5 – firs driven gear (z_2); 6 – second shaft; 7 – second coupling; 8 – third shaft; 9 – second driving gear (z_3); 10 – second driven gear (z_4); 11 – rotary table; 12 – object.

2. OPTIMAL SERVOMOTOR SELECTION CALCULUS ALGORITHM

As previously mentioned, the calculus algorithm includes six major steps [1, 4]:

- 1. defining the working cycle motion diagram for the operated NCA;
- 2. identifying the static and dynamic forces applied to the driven system during operation;
- 3. identifying the mechanical structure on NCA (i.e. specific transfer ratio of each included mechanisms) and preliminary selection of the driving servomotor by checking the kinematic criterion;
- 4. determining of the total resistant equivalent load applied to the driven element;
- 5. secondary selection of the driving servomotor by checking the static and dynamic criterion;
- 6. tertiary selection of the driving servomotor by checking the performance parameters related to full kinematic chain driving (acceleration time, braking time, servomotor's thermal behavior).

The motor finally chosen represents the optimum solution for driving any NCA in terms of satisfying all imposed criteria (kinematic criterion, static criterion, dynamic criterion) and complying as well the necessary performance requirements.

2.1. Step 1. Defining the working cycle motion diagram for the operated NCA

The working cycle motion diagram represents the kinematic basis for the operated NCA. It includes all specific kinematic requirements for the driven NCA (proposed kinematic objectives for both the mechanical structure and the electrical driving system) accordingly application specificity (rotation / translation motion as well NCA integration into NCMT / IR's general assembly). For this purpose, the working cycle motion diagram is usually elaborated as a speed - time diagram including specific motion profiles and kinematic parameters values for all motion phases included in the working cycle.

For each phase of the working cycle a motion profile need to be specified and specific kinematic parameters defined. The most used motion profile for driven elements kinematics characterization is the trapezoidal profile. It includes three motion segments: the first one for driven element's acceleration, the second for it's constant speed motion travel and the third one for driven element's deceleration.

From this point of view, Fig. 5 presents a working cycle motion diagram specifically for a linear motion NCA included in an pick and place IR. The specific working cycle motion diagram includes three phases: phase 1 and phase 2 participating both to the forward direction travel but generating partial strokes characterized by different constant speed values ($v_{\rm I}$, $v_{\rm II}$) and total travel times ($t_0 + t_1 + t_2$ and $t_3 + t_4 + t_5$) and respectively phase 3 for a single reverse direction travel characterized by a third different constant speed value ($v_{\rm III}$) and total time travel ($t_6 + t_7 + t_8$). For each phase, acceleration and deceleration times are equal ($t_0 = t_2$, $t_3 = t_5$, $t_6 = t_8$) and set by taking account the driving system's capability to perform these transitory operations. The total length of forward travel resulting by cumulated motion travel for the



Fig. 5. Sample of working cycle motion diagram including three motion phases (I, II, III), forward direction travel (blue arrow mark) and reverse direction travel (red arrow mark).





first six motion segments (t_0 , t_1 , ..., t_5) is the same with the total reverse travel resulting by cumulated motion for the last three motion segments (t_6 , t_7 , t_8). Total forward travel (for phase 1 + phase 2) and reverse travel (for phase 3) represents the programmed maximum travel length to be reach by driven element.

2.2. Step 2. Identifying the static and dynamic forces applied to the driven system during NCA operation

Considering the working cycle motion diagram presented in Fig. 5, next step in the calculus algorithm is to identify and represent on the motion diagram (Fig. 6) the friction forces (F_f – green marks in fig. 6) and respectively the inertia forces $(F_i - \text{blue marks in Fig. 6})$ applied to the driving system (acting on the driven element as well as generated by the rest of components included in the mechanical structure of the NCA). In Fig. 6 the friction and inertia forces are represented for each motion segments of the three motion phases: phase 1 - forward high speed positioning through the characteristic point of picking-up the manipulated object; phase 2 - transporting forward the picked - up object, by reduced speed, through the end point position; phase 3 reverse travel from the end point position to the start point position, by high speed positioning after object deposition.

Accordingly the specific application that includes the NCA, loads acting on the driving system may vary, some specific aspects being necessary to consider [1]:

 for measuring / dimensional control / inspection or as well non conventional machining (laser / plasma / high pressure water jet etc.) the inertial loads applied to NCA's driving system a related only to inertial forces generated by IR's own components as well as IR's specific end-effectors;

- differently as previously case, in pick and place applications, (machine tending, transfer / transport, assembly, palletizing etc.), the external loads applied to the driving system varies between phases 1 / phase 3 and respectively phase 2, due to manipulate object's weight / inertia that need to be considered additionally to own IR's components weight / inertia.
- for IR's machining applications (by self driven tools end-effectors) or NCMT applications, supplementary to the above mentioned friction forces and inertial forces, in determining the total loads applied to NCA's driving system external forces resulting from machining process need to be considered.

2.3. Step 3 Identifying the mechanical structure on NCA and preliminary selection of the driving servomotor by checking the kinematic criterion

For selecting the appropriate electric servomotor the first condition that need to be accomplish is the kinematic criterion that may be express [1] by following equations (1), (2):

$$Y_{OUT} = V_{driven_el}^{n_\max_sm} \ge V_{nec_driven_el}^{\max}$$
(1)

$$Y_{OUT} \ge V_{nec_driven_el}^{\max} , \qquad (2)$$

where,

- V^{n_max_sm}_{driven_el} is the maximum speed of the driven element that may be obtained when the servomotor is functioning on its maximum available speed and
- $V_{nec_driven_el}^{\max}$ is the maximum necessary speed of the driven element imposed by application specificity.

In order to determe Y_{OUT} the transfer equation (3) for the full kinematic driven chain need to be expressed as follows:

$$Y_{OUT} = Y_{IN} \cdot i_{T_{KC}} , \qquad (3)$$

where, Y_{IN} the input motion, Y_{OUT} the output motion and $i_{T_{KC}}$ is the total transfer ratio for the overall kinematic chain associated to NCA's specific mechanical structure.

However, in equation (3) accordingly the specific NCA's type and structure following differences may appear:

case 1: the input motion Y_{IN} (4) is supplied by a rotary electric servomotor and the output motion Y_{OUT} (5) is obtained as a translation motion of a "carriage / slide" driven element (specific NCA's structures previously presented in Figs. 1 and 2)

$$Y_{IN} = rotation \leftrightarrow Y_{IN} = \overline{\varpi}_{nec_SM}^{\max} , \qquad (4)$$

$$Y_{OUT} = translation \leftrightarrow Y_{OUT} = V_{nec_driven_el}^{\max}, \quad (5)$$

• case 2: the input motion Y_{IN} (6) is supplied by a rotary electric servomotor and the output motion Y_{OUT} (7) is obtained as a rotation motion of a "rotary table" driven element (accordingly specific NCA's structures previously presented in Figs. 3 and 4)

$$Y_{IN} = rotation \leftrightarrow Y_{IN} = \overline{\mathfrak{m}}_{nec}^{\max} S_M, \qquad (6)$$

$$Y_{OUT} = rotation \iff Y_{OUT} = \overline{\varpi}_{nec}^{\max} driven el, \qquad (7)$$

For both above cases the total transfer ratio for the overall kinematic chain associated to NCA's specific mechanical structure, $i_{T_{\kappa c}}$, may be calculated by following equation (8):

$$i_{T_{\kappa c}} = i_1 \cdot i_2 \cdot i_3 \cdot \dots \cdot i_{TM} , \qquad (8)$$

where: $i_1 \cdot i_2 \cdot i_3 \cdot ...$ represent the partially transfer ratio of each subassembly / transmission mechanism included in NCA'a mechanical structure, as may be a planetary gear box (9), a harmonic drive gear box (10) or a cycloidal drive gearbox (11), as well as a spur gear transmission (12) or worm gear transmission (13),

$$i_1 = i_{Planetary_gear_box}, \qquad (9)$$

$$i_1 = i_{Harmonic \ drive} \,, \tag{10}$$

$$i_1 = i_{Cycloidal_drive}, \qquad (11)$$

$$i_1 = i_{Spur_gear} = \frac{z_1}{z_2},$$
 (12)

$$i_1 = i_{Worm_Gear} = \frac{k_W}{z_{WG}},$$
(13)

and i_{TM} represents the transfer ratio of the motion transforming (conversion from rotation into translation) mechanism, which usually may be a ball screw-bearings (14) or a rack and pinion mechanism (15):

$$i_{TM} = \frac{l}{2 \cdot \pi},\tag{14}$$

$$i_{TM} = \frac{m \cdot z_p}{2} \,. \tag{15}$$

As result, the transfer equations for NCA's mechanical structures previously presented in Figs. 1–4 may be rewritten as follows (16):

$$V_{nec_driven_el}^{\max} = \overline{\varpi}_{nec_SM}^{\max} \cdot \frac{z_1}{z_2} \cdot \frac{p}{2 \cdot \pi}$$

$$V_{nec_driven_el}^{\max} = \overline{\varpi}_{nec_SM}^{\max} \cdot \frac{z_1}{z_2} \cdot \frac{m \cdot z_p}{2}$$

$$\overline{\varpi}_{nec_driven_el}^{\max} = \overline{\varpi}_{nec_SM}^{\max} \cdot \frac{z_1}{z_2} \cdot \frac{z_3}{z_4} \cdot \frac{z_5}{z_6}$$

$$\overline{\varpi}_{nec_driven_el}^{\max} = \overline{\varpi}_{nec_SM}^{\max} \cdot \frac{z_1}{z_2} \cdot \frac{z_3}{z_4}$$
(16)

and the maximum necessary speed of the servomotor, can be determined as follow (17):

$$\boldsymbol{\varpi}_{nec_SM}^{\max} = \frac{V_{nec_driven_elem}^{\max}}{\frac{z_1}{z_2} \cdot \frac{p}{2 \cdot \pi}},$$

$$\boldsymbol{\varpi}_{nec_SM}^{\max} = \frac{V_{nec_driven_elem}^{\max}}{\frac{z_1}{z_2} \cdot \frac{m \cdot z_p}{2}},$$

$$\boldsymbol{\varpi}_{nec_SM}^{\max} = \frac{\boldsymbol{\varpi}_{nec_driven_el}^{\max}}{\frac{z_1}{z_2} \cdot \frac{z_3}{z_4} \cdot \frac{z_5}{z_6}},$$
(17)

$$\varpi_{nec_SM}^{\max} = \frac{\varpi_{nec_driven_el}^{\max}}{\frac{z_1}{z_2} \cdot \frac{z_3}{z_4}}$$

Finally, taking in account the motor speed conversion, equations (18) it may be calculated the necessary maximum servomotor speed as follows:

$$\varpi_{nec_SM}^{\max} = \frac{2 \cdot \pi \cdot n_{nec_SM}^{\max}}{60},$$

$$\varpi_{nec_SM}^{\max} = \frac{\pi \cdot n_{nec_SM}^{\max}}{30},$$

$$n_{nec_SM}^{\max} = \frac{30 \cdot \varpi_{nec_SM}^{\max}}{\pi}.$$
(18)

and it may be selected, from the servomotor's catalog, the appropriate servomotor for driving the analized NCA, by applying the final equation for kinematic criterion cheking (19):

$$n_{nec_SM}^{\max} \le n_{SM_rated_(cat)}^{\max}.$$
 (19)

All servomotors accomplishing equation (19) may be primary selected as being compatible with the kinematic restrictions imposed to the driving system for the analyzed NCA.

Remark: In all above equations, on should consider the following measurement units used for following specific terms:

- [mm/sec] for the linear motion driven element's speed *V*;
- [mm] for the module *m* (in rack and pinion case) or screw lead *l* (for ball screw case),
- [rad/sec] for the angular velocity ω,
- [rot/min] for the servomotor speed *n*.

2.4. Step 4. Determining the resistant equivalent load applied to the driven element in case of vertical / horizontal translation axis

Previously to check servomotor compliance with the static and dynamic criterion, the equivalent forces applied to the linear motion driven elements need to be determined.

The equivalent resistant force statically applied (generated by quasi-static loads applied to the driven element) can be determined [1] using relation (20) in the case of horizontal translation axis, and respectively, using relation (22) in the case of vertical translation axis:

$$F_{ST_H} = F_{f_i} = m_{total} \cdot g \cdot \mu, \qquad (20)$$

$$F_{ST_V} = (F_{f_i} + G_i), \qquad (21)$$

$$F_{ST \ V} = (m_{total} \cdot g \cdot \mu) + (m_{total} \cdot g).$$
(22)

The equivalent resistant force dynamically applied generated by both static and dynamic loads (time-varying loads applied to the driven element) can be determined using relation (24) in the case of horizontal translation axis, and respectively, using relation (26) in the case of vertical translation axis:

$$F_{DYN_{H}} = (F_{f_i} + F_{i_i}) \cdot f_d$$
, (23)

$$F_{DYN_{H}} = (m_{total} \cdot g \cdot \mu + m_{total} \cdot a_{T}) \cdot f_{d}, \qquad (24)$$

$$F_{DYN_V} = (F_{f_i} + F_{i_i} + G_i) \cdot f_d , \qquad (25)$$

$$F_{DYN_V} = (m_{total} \cdot g \cdot \mu + m_{total} \cdot a_T + m_{total} \cdot g) \cdot f_d , \qquad (26)$$

where:

- *F_{f_i}* represents the friction forces in the mobile element's guideways / bearings;
- *F_{i_i}* represents the inertial forces generated by the mobile element and attached object;
- *m* total represents the mobile element and attached object total mass;
- μ represents guideway/bearing's friction coefficient;
- *f_d* represents the global dynamic coefficient that takes into account forces application behavior (shock / vibration) and, as usually, has a value of 1.15–1.25;
- *a_T* represents translation acceleration of the mobile element and attached object.

Knowing the maximum speed required to be achieved by the driven element $V_{nec_driven_el}^{max}$ and acceleration time (usually, in of case IR and NCMT, mobile elements being accelerated in 0.35–0.5 sec), one may determine the maximum translation acceleration of the driven element using the equation (27):

$$a_T = \frac{V_{driven_el}^{\max}}{t_{acc}},$$
 (27)

2.5. Step 5. Checking the static and dynamic criteria

Application of the static and dynamic criteria [1] requires the verification of two inequalities (28) and (29):

$$M_{RED_SM}^{STATIC} \le M_{SM_rated_(cat)}^{NOMINAL},$$
(28)

where:

- $M_{RED_SM}^{STATIC}$ represents the reduced torque generated at driving motor shaft level by the static forces applied to the mobile element,
- *M*^{NOMINAL}_{SM_rated_(cat)} represents the rated nominal torque developed by the servomotor (torque developed by servomotor at constant speed), as specified in the servomotor's catalog, and respectively

$$M_{RED_SM}^{DYNAMIC} \le M_{SM_rated_(cat)}^{PEAK},$$
(29)

where: $M_{RED_SM}^{DYNAMIC}$ is the reduced torque generated at the driving servomotor shaft level by static and dynamic forces applied to the mobile element $(F_{f_i} + F_{i_i} + G_i)$.

The torque $M_{SM_{-}rated_{-}(cat)}^{PEAK}$ is the maximum / peak driving motor torque (starting torque) as specified in the catalog.

2.5.1. Evaluation of the static torque reduced at the engine drive shaft level, and checking the selecting servomotor by static criterion

In order to verify equation (28) first it is necessary to evaluate the reduced static torque at the driving servomotor shaft level $M_{RED_SM}^{STATIC}$ starting up from the static resistant torque determined at the mobile element driving element level $M_{rez_driven_el}^{STATIC}$ and taking into account the specific torques transfer equation for overall kinematic chain (30)[1]:

$$M_{OUT} = M_{IN} \cdot \frac{1}{i_{T_{rr}}}, \qquad (30)$$

where, M_{OUT} and M_{IN} may be replaced as:

$$M_{OUT} = M_{rez_driven_el}^{STATIC} , \qquad (31)$$

$$M_{IN} = M_{RED_SM}^{STATIC} , \qquad (32)$$

thus the transfer equation for torques become

$$M_{rez_driven_el}^{STATIC} = M_{RED_SM}^{STATIC} \cdot \frac{1}{i_{T_{rc}}},$$
 (33)

Now, the equation (33) may be also express as:

$$M_{RED_SM}^{STATIC} = M_{rez_driven_el}^{STATIC} \cdot i_{T_{KC}}, \qquad (34)$$

and the equation (28) representing the static criterion to be accomplish by the selected servomotor may be rewrite as:

$$M_{SM_rated_(cat)}^{NOMINAL} \ge M_{RED_SM}^{STATIC} = M_{rez_driven_el}^{STATIC} \cdot i_{T_{RC}} . \quad (35)$$

For all the servomotors accomplishing the kinematic criterion, equation (35) need to be accomplish too in order to have the static criterion satisfied too.

2.5.2. Evaluation of the dynamic torque reduced at the engine drive shaft level and checking the selecting servomotor by dynamic criterion

In order to verify the dynamic criterion expressed by equation (29) first it is necessary to evaluate the reduced dynamic torque at the driving servomotor shaft level $M_{RED_SM}^{DYNAMIC}$ starting up from the dynamic resistant torque determined at the mobile element driving element level $M_{rez_driven_el}^{DYNAMIC}$ and taking into account the specific torques transfer equation for overall kinematic chain (30), however, this time considering following input output torques [1]:

$$M_{IN} = M_{RED}^{DYNAMIC}, \qquad (36)$$

$$M_{OUT} = M_{rez_driven_el}^{DYNAMIC} , \qquad (37)$$

that allows to rewrite the equation (30) as follows (38):

$$M_{rez_driven_el}^{DYNAMIC} = M_{RED_SM}^{DYNAMIC} \cdot \frac{1}{i_{T_{rrc}}}.$$
 (38)

Now, the equation (38) may be also express as:

$$M_{RED_SM}^{DYNAMIC} = M_{rez_driven_el}^{DYNAMIC} \cdot i_{T_{kc}}$$
(39)

and the equation (29) representing the dynamic criterion

to be accomplish by the selected servomotor may be rewrite as (40):

$$M_{SM_rated_(cat)}^{PEAK} \ge M_{RED_SM}^{DYNAMIC} = M_{rez_driven_el}^{DYNAMIC} \cdot i_{T_{kc}}, \quad (40)$$

where $M_{SM_rated_(cat)}^{PEAK}$ represents the rated pick torque developed by the servomotor (maximum starting torque developed by servomotor during driven element's acceleration / deceleration periods), as specified in the servomotor's catalog.

For all the servomotors accomplishing the kinematic criterion and the static criterion, equation (40) need to be accomplish too in order to have the dynamic criterion also satisfied.

All servomotors accomplishing the kinematic criterion the static criterion and the dynamic criterion may be secondary selected as being compatible with the kinematic restrictions imposed to the driving system for the analyzed NCA.

However, in order to perform calculus for checking the dynamic criterion, it is necessary to preliminary evaluate the reduced dynamic torque at the driving servomotor shaft level $M_{RED_SM}^{DYNAMIC}$ starting up from the dynamic resistant torque determined at the mobile element driving element level $M_{rez_driven_el}^{DYNAMIC}$. For this purpose it is necessary to take into account that $M_{rez_driven_el}^{DYNAMIC}$ is summing both static and dynamic resistant torque effects, as below equation (41) and (42) is showing [1]:

$$M_{rez_driven_el}^{DYNAMIC} = M_{rez_driven_el}^{STATIC+INERTIAL},$$
(41)

$$M_{RED_SM}^{DYNAMIC} = M_{RED_SM}^{STATIC} + M_{RED_SM}^{INERTIAL}, \qquad (42)$$

where, included terms may be evaluated as being:

$$M_{RED_SM}^{STATIC} = M_{rez_driven_el}^{STATIC} \cdot i_{T_{KC}}, \qquad (43)$$

$$M_{RED_SM}^{INERTIAL} = J_{T_{KC}}^{SM_SHAFT} \cdot \varepsilon_{SM_SHAFT} .$$
(44)

However, in determining $M_{RED_SM}^{INERTIAL}$ it is necessary to evaluate the angular acceleration on servomotor shaft level ε_{SM_SHAFT} by mean of equation (45) and as well the total inertial load reduced on servomotor shaft level $J_{T_{\kappa c}}^{SM_SHAFT}$ by taking into account the specific mechanical structure of the kinematic chain associated to the NCA by mean of equation (46), (47), (48) and (49) corresponding to the NCA's associated kinematic chains previously presented in Figs.1–4 [1]:

$$\varepsilon_{SM_SHAFT} = \frac{\Delta \overline{\varpi}_{SM}^{\max}}{\Delta t} = \frac{\overline{\varpi}_{SM}^{\max}}{t_{acc}}, \qquad (45)$$

$$J_{T_{KC}}^{SM-SHAFT} = \left[\left(\frac{l}{2 \cdot \pi} \right)^2 \cdot m_{total} + J_S + J_{C_2} + J_{II} + J_{z_2} \right] \cdot \left(\frac{z_1}{z_2} \right)^2 + J_{z_1} + J_I + J_{C_1} + J_{SM} , \qquad (46)$$

$$J_{T_{KC}}^{SM-SHAFT} = \left[\left(\frac{m \cdot z_p}{2} \right)^2 \cdot m_{total} + J_p + J_{C_2} + J_{II} + J_{z_2} \right] \cdot \left(\frac{z_1}{z_2} \right)^2 + J_{z_1} + J_I + J_{C_1} + J_{SM} , \qquad (47)$$

$$J_{T_{KC}}^{SM-SHAFT} = \left\{ \left[(J_{15} + J_{14} + J_{Z_6}) \cdot \left(\frac{z_5}{z_6}\right)^2 + J_{Z_5} + J_{IV} + J_{C_2} + J_{III} + J_{z_4} \right] \cdot \left(\frac{z_3}{z_4}\right)^2 + J_{z_3} + J_{II} + J_{Z_2} \right\} \cdot \left(\frac{z_1}{z_2}\right)^2 + (48) + J_{Z_2} + J_{I} + J_{C_2} + J_{SM}$$

$$J_{T_{kc}}^{SM_{-}SHAFT} = \left[(J_{12} + J_{11} + J_{Z_4}) \cdot \left(\frac{z_3}{z_4}\right)^2 + J_{Z_3} + J_{III} + J_{C_2} + J_{II} + J_{Z_2} \right] \cdot \left(\frac{z_1}{z_2}\right)^2 + J_{Z_1} + J_{I} + J_{C_1} + J_{SM} .$$
(49)

Remarks for calculus procedure operation:

1. In checking the dynamic criterion iterative calculation need to be performed. From this point of view in determining the $J_{T_{kc}}^{SM_{-}SHAFT}$ it need to take account that J_{SM} value is a specific issue for each tested servomotor, and need to be actualized on each iterative step. That is why, in performing the first iteration for testing available servomotors from the catalog, will be necessary to take account about first servomotor's J_{SM} value and include it in equations (46), (47), (48) or (49). After $J_{T_{kc}}^{SM}$ -SHAFT evaluation, $M_{RED_{SM}}^{INERTIAL}$ may be calculated using equation (44) and considering the value previously determined by equation (43) for $M_{RED_SM}^{STATIC}$, it may be calculated the specific value of $M_{RED_SM}^{DYNAMIC}$ by using equation (42) and finally the dynamic criterion specific equation (40) may be verified. If the condition (40) is not satisfied, the J_{ME} value is replaced with the next J_{ME} value corresponding to the second available motor from the catalog already checked as satisfying the kinematic and static criterion, and the calculation of $J_{T_{KC}}^{SM-SHAFT}$ and the calculus procedure for checking the dynamic criterion accomplishing is performed again. Similarly may be proceed for performing the third and next iterative steps by the accomplishment of the dynamic criterion, the servomotor finally chosen by meeting all three required criterions: kinematic criterion, the test of the static torque by the motor rated nominal torque and the test of dynamic torque by

2. In evaluating $M_{RED_SM}^{INERTIAL}$ by equation (44) it is also necessary to take into account too, that for different motion phases ε_{SM_SHAFT} may have different values, corresponding to different maximum constant speed to be rich by the driven element in the same acceleration / deceleration time). That is why usually a complementary working cycle torque diagram is accomplished in order to correctly evaluate the total resistant torque on each motion segment of all included motion phases (Fig. 7) [1].

the motor rated pick (starting) torque.

That is why a specific total value of Root Mean Square Torque M_{RMS} is usually calculated for each motion phase / overall working cycle as will be following presented.

3. In the mean time it is necessary to specify that also static torque may have different values from different reasons [1].

A first reason for which the static torque may have different values from one motion phase to another, is due to variations of the normal (gravitational) loads applied to the driven element during different motion phases. Such case may be the case of previously discussed IR's working cycle motion diagram's second motion phase (involving the pick-up of a manipulated object) characterized by a greater static torque than the static torque



Fig. 7. Identifying the specific static and dynamic torques on working cycle torque diagram during NCA operation: M_{stl} , M_{stII} , M_{stII} , M_{stIII} , - static torques and M_{dynI} , M_{dynII} , M_{dynIII} , - dynamic

torques, acting in motion phases I, II and III, [1].

corresponding to the first and third phases (as shown in Fig. 7)

A second reason for which the static torque may have different calculus equation than above presented may be the specific mechanical structure of the NCA's kinematic chain. An illustrative sample may be the case of NCA presented in Fig. 1, where supplementary to the static torque due to friction forces acting on guideways / bearings level, the total resistant static torque includes also specific terms for quantifying the resistant static torque components due to ball screw nuts preloading and ball screw bearing sets preloading. A similar case may be too considered for the NCA presented in Fig. 2, where a specific terms for quantifying the resistant static torque component due to pinion shaft's bearings preloading need to be considered [1].

2.6. Step 6. Checking the motor performance parameters for driving the overall kinematic chain

6.1. Checking the performance parameters for servomotor's acceleration time and braking time

After selecting a servomotor from the catalog by checking it's compliance with the above mentioned three criteria (kinematic, static and dynamic), the servomotor's performance parameters, during acceleration and braking motion segments need be investigated [1]. For this purpose as previously mentioned, it is already known that for IR and NCMT the acceleration and braking time are ussually imposed as being limited by following values:

$$0.35 \le t_{acc} \le 0.5 \,\mathrm{sec} \,,$$
 (50)

$$0.35 \le t_{break} \le 0.5 \,\text{sec} \tag{51}$$

Considering above mentioned performance parameters limits, the selected servomotor compliance in terms of acceptable acceleration time and braking time parameter values may be check after specific values of t_{acc} and t_{break} evaluation by using equations (52) and (53) [1]:

$$t_{acc} = \frac{4}{375} \cdot J_{T_{\kappa c}}^{SM_SHAFT} \cdot n_{SME_rated_(cat)}^{\max} \cdot \left(\frac{0.3}{M_{SM_rated_(cat)}^{PEAK} - M_{S/P} \cdot i_1} + \frac{0.7}{\frac{1}{5} \cdot M_{SME_rated_(cat)}^{PEAK} - M_{S/P} \cdot i_1} \right), \quad (52)$$

$$t_{break} = \frac{4}{375} \cdot J_{T_{kc}}^{SM_SHAFT} \cdot \frac{n_{SM_rated_(cat)}^{max}}{M_{SM_rated_(cat)}^{BREAK} - M_{S/P} \cdot i_{1}}.$$
(53)

6.2. Checking the performance parameters for servomotor's thermal behavior

The final checking for driving servomotor optimum selection is related to its thermal behavior evaluation [1, 3, 4].

The calculus algorithm is valuable for the thermal behavior evaluation of both brush / brushless DC servomotors, and as well, by taking account of some specific adjustment coefficients for the AC synchronous servomotors [1, 4].

The servomotor thermal behavior evaluation includes three stages: evaluation of the maximum current applied to servomotor windings, evaluation of the thermal energy dissipated in servomotor windings, determining the maximum overheating temperature of the servomotor and comparison of servomotor maximum temperature with the allowed temperature limit for servomotor safe functioning (catalog rated).

Thus, first step in evaluating the thermal dissipated energy (by Joule effect) in servomotor windings is to calculate the total torque necessary to be supplied by the servomotor in terms of complying with total resistant torque determined by equation (54) [1]:

$$M = M_I + M_f + M_L$$
 [Nm], (54)

where:

- M_I represents the inertial torque necessary for acceleration / deceleration of overall kinematic chain, reduced on the servomotor shaft level,
- $M_{\rm f}$ = friction torque in motor bearings, and other static resistant torques generated by complementary effects (as may be bearings preloading, ball screw nuts preloading etc.) which, in this calculation algorithm, will be considered as negligible;
- M_L representing torque generated by other specific external forces (as may be cutting forces for the case of NCMT's NCA) which, in this calculation algorithm, will be too considered as negligible.

Thus it may be assumed that for current calculus algorithm the total resistant torque applied to the servomotor is due to only inertial torque $M_I(55)$:

$$M = M_I \quad [Nm], \tag{55}$$

Continuing by this assumption it may be now determined the specific value of the current applied to servomotor windings in order for supplying the above necessary torque complying with the resistant torque as follows (56) and (57) [3]:

$$M_I = k_m \cdot I_a \quad [Nm], \tag{56}$$

$$I_a = \frac{M_I}{k_m} [A], \tag{57}$$

where I_a represents the current applied on servomotor windings and k_m represents servomotor's torque constant. As previously illustrated the inertial torque M_I may be expressed as (59) [1, 4]:

$$M_I = J_{T_{\kappa c}}^{SM _SHAFT} \cdot \varepsilon_{SM} , \qquad (59)$$

where, for the case of rotation axis illustrated in fig. 3 and fig.4 $J_{T_{kc}}^{SM_{-SHAFT}}$

may be detailed as (60):

$$J_{T_{kc}}^{SM_{-}SHAFT} = (i_G)^2 \cdot J_l + J_G + J_{ME},$$
(60)

where J_l represents the inertial load of the driven element and need to be evaluated accordingly the specific mechanical structure of the NCA's kinematic chain, J_G represents the inertial load of the gearing mechanism and J_{ME} represents the servomotor's rotor inertia and i_{G} represents the transfer ratio of the included gearing system. However, for the case of a translation axis including a rack and pinion mechanism (fig.2) J_l may be calculated by using equation (61), and for the case of a translation axis including a ball screw mechanism (Fig. 1) J_l may be calculated by using equation (62), [1, 4]:

$$J_l = J_p + m_{total} \cdot i_{TM}^2, \qquad (61)$$

$$J_l = J_S + m_{total} \cdot i_{TM}^{2}.$$
(62)

Taking account of equations (61) and (62) equations (60) may be specifically rewritten as equations (63) and (64) for the case of rack and pinion mechanism, and respectively as equations (65) and (66) for the case of ball screw mechanism:

Thus the final equation for determining the inertial torque become (67) and equation 57 may be rewritten as (68) [1, 4]:

$$J_{T_{KC}}^{SM_SHAFT} = i_{G}^{2} \cdot (J_{p} + m_{total} i_{TM}^{2}) + J_{G} + J_{ME}, \qquad (63)$$

$$J_{T_{kc}}^{SM_SHAFT} = i_G^{-2} \cdot \left[J_P + \left(\frac{m \cdot z_p}{2} \right)^2 \cdot \left(m_{tota} \right) \right] + J_G + J_{ME}$$
, (64)

$$I_{T_{KC}}^{SM_{-}SHAFT} = i_{G}^{2} \cdot (J_{S} + m_{total} \cdot i_{TM}^{2}) + J_{G} + J_{ME}, \quad (65)$$

$$I_{xc}^{SM_SHAFT} = i_G^{2} \cdot \left[J_S + m_{total} \cdot \left(\frac{l}{2 \cdot \pi}\right)^2 \right] + J_G + J_{ME} .$$
(66)

By taking into account above equations the inertial torque and correspondent current applied on servomotor windings may be finally evaluated using equations (67) and (68) [1, 4]:

$$M_I = \left(i_G^2 \cdot J_I + J_G + J_{ME} \right) \cdot \varepsilon_{ME} , \qquad (67)$$

$$I_a = \frac{M_I}{k_m} = \frac{\left(i_G^2 \cdot J_l + J_G + J_{ME}\right) \cdot \varepsilon_{ME}}{k_m} .$$
(68)

6.2.1. Evaluation of thermal energy dissipated in the servomotor windings

In evaluating the thermal energy dissipated in the servomotor windings it is assumed that for a working cycle motion diagram as presented in Fig. 5, in each motion phase the acceleration time and the deceleration time are equal.

Following this assumption, the thermal energy dissipated in the servomotor windings by Joule effect may be calculated as a function of I_a and R_a and the total cycle time t_c by using equation (69). In equation (69), as previously presented the value of I_a may be evaluated by equation (68) as depending by total inertial loads and the transfer ratio of the included gearing system, and the R_a represents the total electrical resistance of servomotor windings [3].

$$W_d = \int_0^{t_c} (I_a^2 \cdot R_a) dt \cdot$$
(69)

Considering the three motion phases of the working cycle, equation (69) may be rewrited as (70) [1, 4]:

$$W_{d} = \int_{0}^{t_{1}} (I_{a}^{2} \cdot R_{a}) dt + \int_{t_{1}}^{t_{2}} (I_{a}^{2} \cdot R_{a}) dt + \int_{t_{2}}^{t_{3}} (I_{a}^{2} \cdot R_{a}) dt, \quad (70)$$

where, each term may be further developed as equations (71), (72) and (73) [1, 4]:

$$\int_{0}^{t_1} (I_a^2 \cdot R_a) dt = \int_{0}^{t_{ard}} (I_a^2 \cdot R_a) dt + \int_{t_{ard}}^{t_{e,cd}} (I_a^2 \cdot R_a) dt + \int_{t_{e,cd}}^{t_{bd}} (I_a^2 \cdot R_a) dt, \quad (71)$$

$$\int_{t_1}^{t_2} (I_a^2 \cdot R_a) dt = \int_{t_1}^{t_{ax^2}} (I_a^2 \cdot R_a) dt + \int_{t_{ax^2}}^{t_{ax^2}} (I_a^2 \cdot R_a) dt + \int_{t_{ax^2}}^{t_{ax^2}} (I_a^2 \cdot R_a) dt, \quad (72)$$

$$\int_{t_2}^{t_1} (I_a^2 \cdot R_a) dt = \int_{t_2}^{t_{are3}} (I_a^2 \cdot R_a) dt + \int_{t_{are3}}^{t_{r_r,cr3}} (I_a^2 \cdot R_a) dt + \int_{t_{r_r,cr3}}^{t_{hr3}} (I_a^2 \cdot R_a) dt \cdot (73)$$

Considering too equation (60), after evaluation of each term, the total thermal energy dissipated in servomotor windings may be determined by equations (73) and (74) [1, 4]:

$$W_{d} = \left[\left(2 \cdot \frac{1}{i_{G}}\right) \cdot \frac{\left[i_{G}\right]^{2} \cdot J_{I} + J_{G} + J_{ME}}{k_{m}} \cdot \left(\frac{\overline{\omega}_{p/s1}}{t_{a1}} + \frac{\overline{\omega}_{p/s2}}{t_{a2}} + \frac{\overline{\omega}_{p/s3}}{t_{a3}}\right)^{2} \cdot R_{a}, \quad (73)$$

$$W_{d} = 4 \cdot R_{a} \cdot \frac{1}{i_{G}^{2}} \cdot \frac{1}{k_{m}^{2}} \cdot \frac{|t_{G} \cdot J_{l} + J_{G} + J_{ME}|}{1} \cdot \left(\frac{\omega_{p/s1}}{t_{a1}} + \frac{\omega_{p/s2}}{t_{a2}} + \frac{\omega_{p/s3}}{t_{a3}}\right)$$
(74)

and, if i_G^2 is assumed as being noted by y (75) by successively transformations the final equation (76) may be determined:

$$W_{d} = 4 \cdot R_{a} \cdot \frac{1}{y} \cdot \frac{1}{k_{m}^{2}} \cdot \frac{(y \cdot J_{l} + J_{RED} + J_{m})^{2}}{1} \cdot \left(\frac{\varpi_{p/s1}}{t_{a1}} + \frac{\varpi_{p/s2}}{t_{a2}} + \frac{\varpi_{p/s3}}{t_{a3}}\right)^{2}, (75)$$
$$W_{l} = \frac{4R_{a}}{k_{m}^{2}} \left[(J_{G} + J_{ME})^{2} \cdot \frac{1}{y} + 2\frac{(J_{G} + J_{M})^{2}(J_{l})}{1} + \frac{y \cdot J_{l}^{2}}{1} \right] \left(\frac{\varpi_{p/s1}}{t_{a}} + \frac{\varpi_{p/s2}}{t_{a}} + \frac{\varpi_{p/s3}}{t_{a}}\right)^{2}. (76)$$

On the other side, it is known that the thermal energy dissipated by Joule effect is dependent by square of the current applied to servomotor windings. The current is depending itself (by mean of torque constant) of inertial torque M_l , that depends itself by square of reducer gearing transmission ratio. Thus it may be assumed that the thermal energy dissipated by Joule effect is dependent by square of reducer gearing transmission ratio, as equation (77) is shown [1, 3, 4]:

$$W_d = f(y) = f(i_G^2).$$
 (77)

However, if the target of determining the minimum dissipated thermal energy in servomotor windings is assumed, a derivative process may be performed for equation (76) in order to determine the optimum transfer ratio value i_{G_opt} of the ideal gearing system corresponding to this target, as equation (78), (79), (80) and (81) are shown [1, 4]:

$$\frac{dW_d}{dy} = 0, \qquad (78)$$

$$0 = (J_{ME} + J_G)^2 \cdot \left(\frac{-1}{y^2}\right) + J_l^2,$$
(79)

$$y^{2} = \frac{(J_{ME} + J_{G})^{2}}{J_{l}^{2}},$$
 (80)

$$\frac{\left(J_{ME}+J_{G}\right)}{J_{l}}=i_{G_opt}.$$
(81)

By replacing the determined value of optimal gearing transfer ratio (81) into equation (76) it may be determined now the minimum thermal dissipated energy in servomotor windings, as equation (82) is shown:

$$W_{d} = 16 \frac{R_{a}}{k_{m}^{2}} \cdot J_{l} \cdot (J_{G} + J_{ME}) \cdot \left(\frac{\overline{\omega}_{p/s1}}{t_{a1}} + \frac{\overline{\omega}_{p/s2}}{t_{a2}} + \frac{\overline{\omega}_{p/s3}}{t_{a3}}\right)^{2}.$$
 (82)

Successively, for evaluating the thermal behavior of the servomotor, the internal and external temperature need to be determined by using equations (83) and (84) [5]:

$$T_{2_INT} = \frac{I_{RMS}^{2} \cdot R_{a0_SM} \cdot R_{t_SM} \cdot (1 - 0.00393 T_{0}) + T_{0}}{1 - 0.00393 R_{a0_SM} \cdot R_{t_SM}}, \quad (83)$$

$$T_{2_EXT_SM} = T_{2_INT} \cdot R_{t_SM} .$$
(84)

Where I_{RMS} represents root mean square current determined by equation (85) and M_{RMS} , represents root mean square torque evaluated by equation (86) [5], $R_{t_{SM}}$ represents the thermal constant of the servomotor, and

 R_{a0} represents servomotor winding's electrical resistance at the reference temperature of $T_0=20$ °C.

$$I_{RMS} = \frac{M_{RMS}}{k_m} + I_O, \qquad (85)$$

$$M_{RMS} = \sqrt{\frac{M_{accelerare}}^{2} \cdot t_{acc} + M_{Vct}^{2} \cdot t_{Vct} + M_{franare}^{2} \cdot t_{franare}}{t_{acc} + t_{Vct} + t_{franare}}} .(86)$$

However, it is necessary to note that in evaluating servomotor thermal behavior in equation (83) the internal temperature of servomotor windings is determined as a function of R_{a0} . In the mean time, if ambient temperature is different than $T_0=20$ °C or the servomotor has already previously run and warmed, it need to take account that servomotor winding's electrical resistance is varying proportionally with the temperature increasing, thus the specific winding electrical resistance R_{a2} at T_{2_INT} need to be reevaluated by using equations (87) [5]:

$$R_{a2} = R_{a0 \ ME} \cdot [1 + 0.00393 \cdot (T_{2 \ INT} - T_{0})].$$
(87)

Thus, having determined the winding electrical resistance R_{a2} the to T_{2_INT} temperature and replacing R_{a0} from equation (83) by R_{a2} a new T_{2_INT} may be determined, and further the external temperature of the servomotor T_{2_EXT} reevaluated by equation (84).

Finally the total servomotor internal temperature's increasing Δt_{INT} and Δt_{INT} may be evaluated and the effective maximum servomotor internal temperature t_{MAX} _ INT _ EFF _ SM compared with rated allowable servomotor operation temperature t_{SM} _ MAX _ ALLOWED

$$t_{MAX _ INT _ EFF _ SM} =$$

$$t_{ENVIRONMEN} + \Delta t_{INT_SM} \le t_{MAX_INT_SM_ALLOWED} , (88)$$

where $t_{MAX_INT_SM_ALLOWED}$ is representing the maximum allowable internal temperature for servomotor operation (specified in servomotor catalog) and usually $t_{MAX_INT_SM_ALLOWED} \le 130^{\circ}...150^{\circ}C$.

In case of over passing the admissible limit for servomotor's internal temperature a successive (increased size) servomotor need to be selected and the thermal behavior evaluation of the new selected servomotor is performed similarly as previously presented, by taking into account appropriate new input data for the new selected servomotor [1, 4].

7. CONCLUSIONS

The paper presented a complete calculus algorithm for selecting the optimal servomotors of the kinematic chains included in the numerically controlled axes (NCA) of machine tools (NCMT) and industrial robots (IR). The algorithm can be applied for both type controlled axes, i.e. IR, and NCMT, as well as either rotation axis or translation axis, regardless of their mechanical structure. The presented calculus algorithm includes six major steps and allows to select the optimum solution for electric driving of the NCA in terms of satisfying all imposed criteria (kinematic criterion, static criterion, dynamic criterion) and complying as well the necessary performance requirements for NCA's electromechanic driving system performances (acceleration / braking time and servomotor optimum thermal behavior).

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