GRAPHICAL METHOD IN CATIA ENVIRONMENT FOR PROFILING ROTATING CUTTERS

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Abstract: *In this paper, we present a graphical method, developed in CATIA environment, based on the method of substituting circles family and dedicated to profiling rotating cutters. Hereby, the family of substituting circles associated with the worked piece (the profile to be generated) is determined at the same time to the curve transposed, in the rolling motion, to the tool centrod. After finding the position of the contact points between the generated profile and the family circles, we can determine, in the transposed family, the locus of the points from the reciprocal enwrapping profile – the rotating cutter profile. The paper also includes a method application for generating the helical surface of a ball screw, and a comparison between the profiling results when using the graphical method, versus an analytical method. The results prove the coincidence between the two profiles determined as above, on one hand, and both rapidity and precision of the graphical method, on the other hand.*

Key words: *graphical method, CATIA, reciprocal enwrapped surfaces, rotating cutter, ball screw.*

1. INTRODUCTION

The rotating cutters are cutting tools that generate by enwrapping. They use the rolling method, applied to complex revolution surfaces with periodic axial profile, mostly represented by cylindrical helical surfaces with constant pitch.

In practice, the cutting tools of rotating cutter type are used to generate, by turning, the ball-screw axial profile. Another possible application regards globoidal worms machining.

The profiling methods for this type of tools ground on the general principles of surfaces enwrapping (in the plane problem case – profiles enwrapping), by starting from Gohman fundamental theorem [1, 2].

Complementary analytical methods, like Family of Substitutive Circles Method [3], Minimum Distance Method [4] or Plain Generating Trajectories Method [5] are alternative solutions to solve the same problem.

On one hand, the analytical methods have the advantage of being universal and enabling to obtain, both rigorously and rapidly, the co-ordinates of the points from the rotating cutter profile. On the other hand, the analytical method application is not intuitive enough and, for this reason, the numerical errors possible to occur are hardly interpretable.

In this paper, we suggest a graphical method, developed in CATIA designing environment [6, 7], based on the method of substituting circles family and dedicated to

profile rotating cutters. This method has the important advantage of being intuitive and, therefore, the profiling errors can be eliminated.

2. THE SUBSTITUTING CIRCLES FAMILY METHOD

The following reference systems are necessary, according to the notations from Fig. 1:

- *xy*, meaning the global system, having the origin in the center of the centrod C_2, of the rotating cutter;

- *XY* – relative system, associated to the axial section of the screw to be generated;

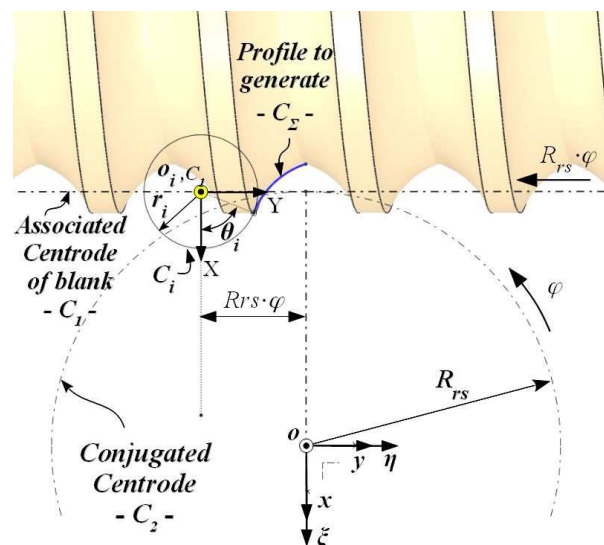- *ξη* – relative system, attached to the rotating cutter, during its motion.

Fig. 1. The centrods associated to profiles. Reference systems.

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The first step of the substituting circles family method requires defining the circles family, having their centers on C_1 centrode and being, simultaneously, tangent to the screw profile from its axial section, C_Σ.

The current circle of the circles family, having its center in the point O_i, is defined, in the xy system by the equations:

$$(C_i)\begin{vmatrix} X = -r_i \cdot \cos\theta_i; \\ Y = R_{rs} \cdot \varphi - r_i \cdot \sin\theta_i, \end{vmatrix} \quad (1)$$

with r_i and θ_i meaning the family variables, while φ is an arbitrary variable.

The axial section of the screw to be generated (C_Σ profile) is considered as known and defined through equations of the type:

$$C_\Sigma \begin{vmatrix} X = X(u); \\ Y = Y(u), \end{vmatrix} \quad (2)$$

with u – variable.

The tangency conditions between the two curves, (1) and (2), are the following:

$$\begin{cases} X(u) = -r_i \cdot \cos\theta_i; \\ Y(u) = R_{rs} \cdot \varphi - r_i \cdot \sin\theta_i; \\ \dot{X}_u = r_i \cdot \sin\theta_i; \\ \dot{Y}_u = -r_i \cdot \cos\theta_i. \end{cases} \quad (3)$$

The tangency conditions (3) determine both the dependence of r_i and θ_i, on the variables φ and u and the specific enveloping condition – $\varphi = \varphi(u)$.

The rotating cutter profile results by transposing the (C_i) family of circles, by rolling, on C_2 centrode of the rotating cutter. The envelope of this circle family means the rotating cutter profile, Fig. 2.
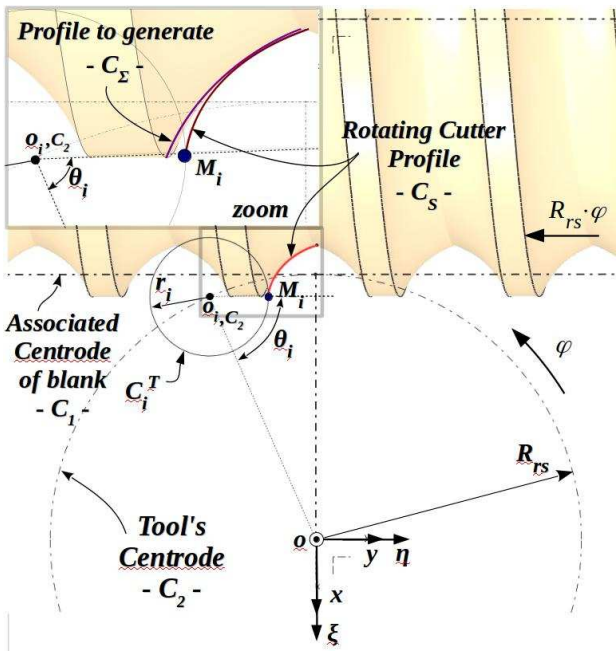
3. CATIA GRAPHICAL METHOD

The graphical method that we have developed turns to profit the facilities of CATIA graphical design environment and presume to follow some successive steps, according to the "Substituting Circles Family" method, as presented below (see also Fig. 3).

- In the beginning, the feet of the normals drawn into the points A and B are determined on y axis – the C_1 centrode, as the points a and b, respectively, by using the command *Tangent* from *Sketch* module.

- The $\overline{ab}$ segment is then divided into n equal or arbitrarily chosen parts, depending on the profile shape. This way, the circles centers $O_{i-1\ C_1}$, $O_{i\ C_1}$, $O_{i+1\ C_1}$ are resulting, for an arbitrarily chosen angular increment $\Delta\varphi$,

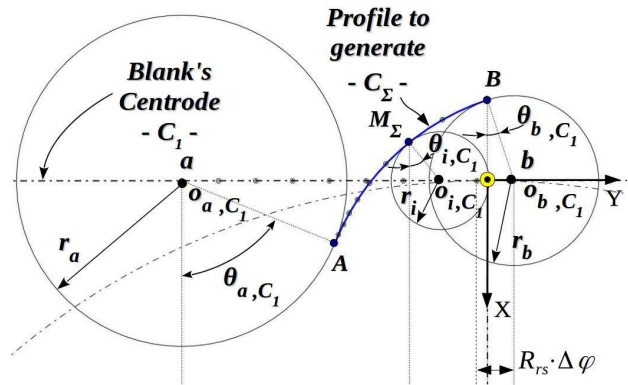$$\overline{O_{iC_1}O_{i+1C_1}} = \frac{\overline{ab}}{n}. \quad (4)$$
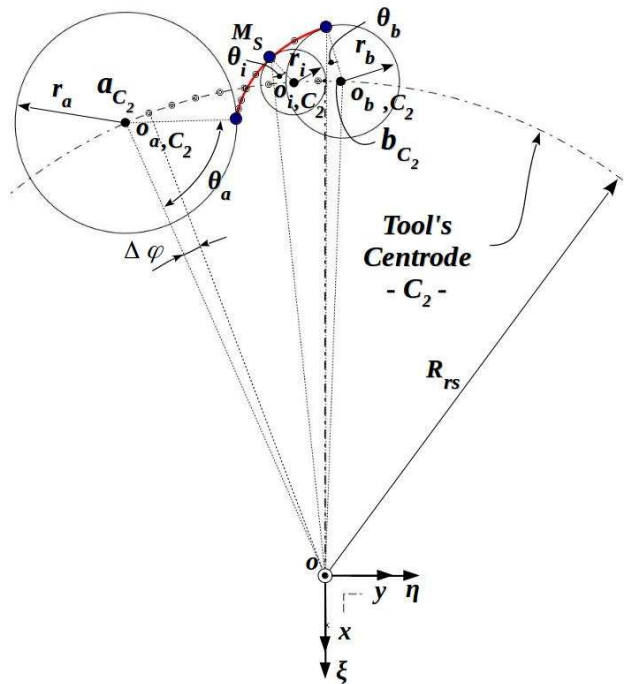

Fig. 3. The substituting circles family.

Fig. 2. The substituting circles family transposed onto the rotating cutter centrod.

Fig. 4. The circles family transposed onto C_2 centrode.

- The circles having their centers into the O_i points and tangent to C_Σ profile are further drawn (command *Tangent*). The r_i radii and the θ_i angles, corresponding to M_Σ tangency points can, now, be measured.

- The arcs of circle $O_{iC_2}O_{i+1C_2}$, having equal length to $\overline{O_{iC_1}O_{i+1C_1}}$ segments are measured onto the R_{rs} circle – the C_2 centrod – hence finding the centers of the transposed circles family (see Fig. 4).

- The circles having their centers into the O_{iC2} points can, now, also be drawn and the points M_S, similar to M_Σ points, can be determined.

- The ensemble of all the M_S points represents the rotating cutter profile.

4. NUMERICAL APPLICATION: PROFILING OF THE ROTATING CUTTER FOR BALL SCREW MACHINING

The rotating cutter profiling was realized, successively, by using two different methods: the graphical method, introduced by this paper, and an analytical method (the Plain Generating Trajectories Method [5]). The second method application was performed in order to create the possibility of assess the correctness and the precision of the graphical method.

4.1. Graphical method application

We further present an example of applying the graphical method, in order to profile a rotating cutter for generating the helical surface of a ball screw (Fig. 5). From symmetry reasons, only the left flank of the ball screw profile was considered.

The ball screw profile considered in the application is defined by the following geometrical parameters:

- Profile radius – $R = 16$ mm;
- Co-ordinates of the circle defining the axial profile – $e = 5$ mm; $n = 5$ mm;
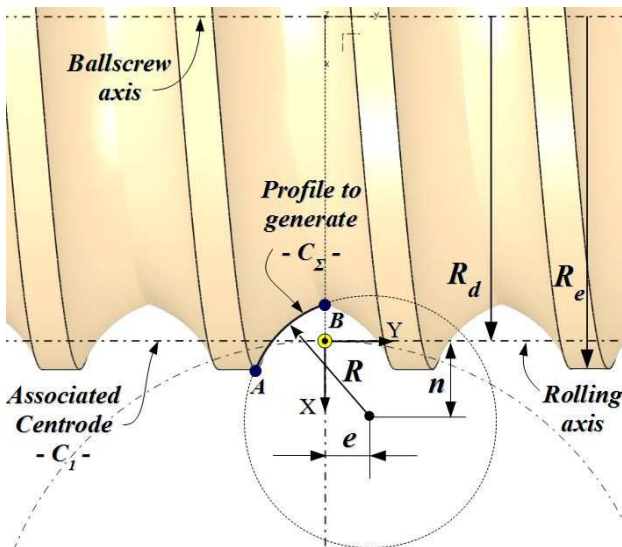- Ball screw exterior radius – $R_e = 47$ mm.

Fig. 5. Geometry of the ball screw axial section.

The radius of the two centrods, associated to the worked piece and to the rotating cutter are $R_d = 46$ mm and $R_{rs} = 47.746$ mm, respectively.

The graphical method has been used according to the steps mentioned in Section 3. The $\overline{ab}$ interval (from relation (4)) was divided into $n_i = 100$ equal parts. The centers of the circles owning to the family transposed on C_2 centrod, O_{iC2}, were determined for the angular increment

$$\Delta\varphi = \frac{\overline{O_{iC_1}O_{i+1C_1}}}{R_{rs}}. \tag{5}$$

The co-ordinates of the resulted tool profile have been measured in 100 points, some of them being mentioned in Table 1.

4.2. Plain Generating Trajectories method application

Plain Generating Trajectories method application started from the equations of the ball screw axial profile (see notations from Fig. 5):

$$C_\Sigma \begin{vmatrix} X = n - R \cdot \cos\theta; \\ Y = e - R \cdot \sin\theta. \end{vmatrix} \tag{6}$$

The generation process kinematics is reflected, if considering the reference systems and the notations already introduced at the beginning of section 1 (see also Fig. 1), by the equation:

$$\xi = \omega_3(\varphi)[X + a], \tag{7}$$

where ω_3 is a matrix of rotation co-ordinates transform,

$$\omega_3(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

φ meaning a variable angular parameter, and

$$a = \begin{pmatrix} -R_{rs} \\ -R_{rs} \cdot \varphi \\ 0 \end{pmatrix}, \tag{9}$$

R_{rs} meaning the tool rolling radius.

If in the equation (7), the matrix X contains the co-ordinates of the current point from the generated profile, (6), then by making this replacement and after calculus, we obtain the equations of the C_Σ profiles family in the relative motion up against the tool reference system:

$$(C_\Sigma)_\varphi \begin{vmatrix} \xi = [(n - R \cdot \cos\theta) - R_{rs}]\cos\varphi + \\ + [(e - R \cdot \sin\theta) - R_{rs} \cdot \varphi]\sin\varphi; \\ \eta = -[(n - R \cdot \cos\theta) - R_{rs}]\sin\varphi + \\ + [(e - R \cdot \sin\theta) - R_{rs} \cdot \varphi]\cos\varphi. \end{vmatrix} \tag{10}$$

with θ and φ – variable parameters. The general form of equations (10) is, therefore:

$$(C_\Sigma)_\varphi \begin{vmatrix} \xi = \xi(\theta, \varphi); \\ \eta = \eta(\theta, \varphi). \end{vmatrix} \qquad (11)$$

The enveloping condition, according to Plain Generating Trajectories method [5] has, in this case, the specific form:

$$\frac{\xi'_\theta}{\xi'_\varphi} = \frac{\eta'_\theta}{\eta'_\varphi}, \qquad (12)$$

The derivatives of the functions from the above relation can be calculated from the equations (10); their resulting expressions are:

$$
\begin{aligned}
\xi'_\theta &= R \cdot \sin\theta \cdot \cos\varphi - R \cdot \cos\theta \cdot \sin\varphi = \\
&= R \cdot \sin(\theta - \varphi); \\
\eta'_\theta &= -R \cdot \sin\theta \cdot \sin\varphi - R \cdot \cos\theta \cdot \cos\varphi = \\
&= -R \cdot \cos(\theta - \varphi); \\
\xi'_\varphi &= -(n - R \cdot \cos\theta)\sin\varphi + \\
&\quad + (e - R \cdot \sin\theta - R_{rs} \cdot \varphi)\cos\varphi; \\
\eta'_\varphi &= -(n - R \cdot \cos\theta)\cos\varphi + \\
&\quad + (e - R \cdot \sin\theta - R_{rs} \cdot \varphi)\sin\varphi;
\end{aligned}
\qquad (13)
$$

After substituting (13) in (12), a relation of dependence, having the general form:

$$\varphi = \varphi(\theta), \qquad (14)$$

can be obtained.

The ensemble formed by the equations (10) and the relation (14) gives the envelope of C_Σ profiles family, meaning the rotating cutter profile.

The numerical application of the Plain Generating Trajectories method, in order to solve the profiling problem approached here, firstly requires finding the variation limits of θ angular parameter corresponding to the length of the generated profile (see Fig. 5):

$$\theta_{min} = \frac{\pi}{2} - \arccos\frac{e}{R} = \arcsin\frac{e}{R} \qquad (15)$$

and

$$\theta_{max} = \frac{\pi}{2} - \arcsin\frac{n}{R} = \arccos\frac{n}{R}. \qquad (16)$$

A MATLAB application has been developed in order to implement this profiling method. The input data are:
- The radius R and the center co-ordinates (e, n) of the arc of circle defining the ball-screw profile in axial section (see Fig. 5);
- The radii of the two centrods, R_d and R_{rs};
- The number of points for defining the rotating cutter profile, n_p.

The results delivered by the application are the co-ordinates of the points from both screw ball and rotating cutter profiles. The results obtained by running the application for the same values of the input parameters as used in the case of the graphical method are selectively presented in Table 1.

Table 1

Co-ordinates of the points from the rotating cutter profile

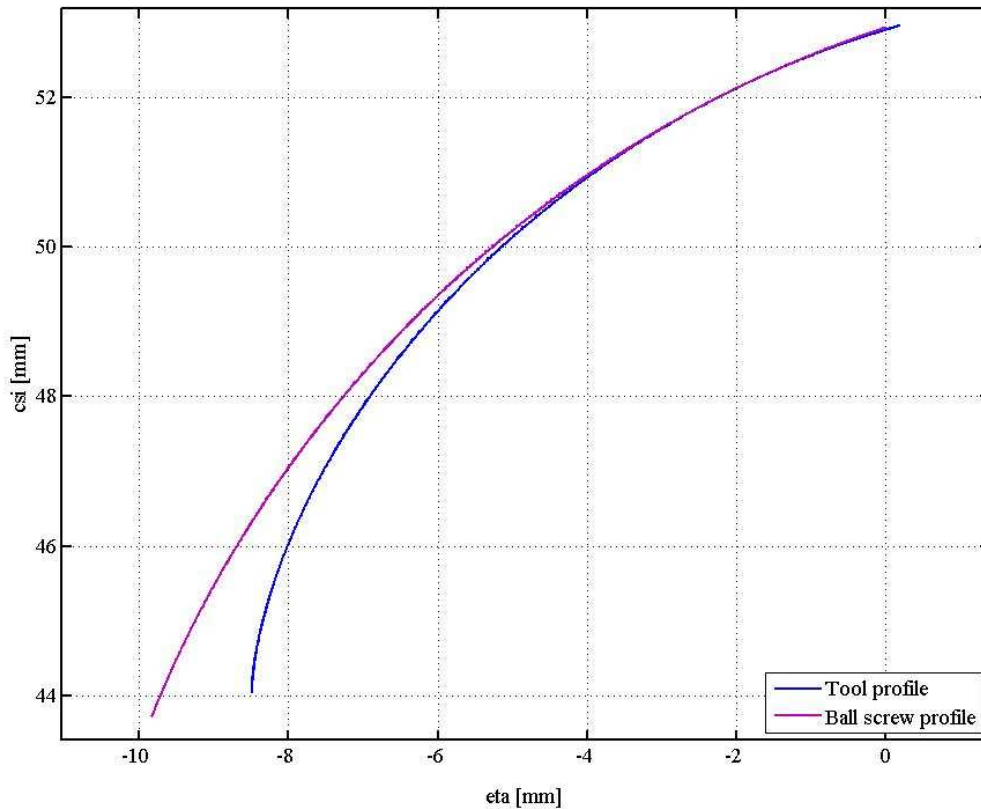| Point crt no	Analytical method		Graphical method	
	ξ [mm]	η [mm]	ξ [mm]	η [mm]
1	-52.9719	0.1868	-52.9720	0.1870
2	-52.9256	0.0304	-52.9350	0.0590
3	-52.8775	-0.1254	-52.8960	-0.0680
4	-52.8275	-0.2807	-52.8560	-0.1950
5	-52.7758	-0.4354	-52.8150	-0.3210
6	-52.7222	-0.5895	-52.7720	-0.4470
7	-52.6668	-0.7429	-52.7290	-0.5730
8	-52.6097	-0.8957	-52.6840	-0.6980
9	-52.5507	-1.0478	-52.6380	-0.8230
10	-52.4900	-1.1992	-52.5910	-0.9470
11	-52.4275	-1.3499	-52.5420	-1.0710
12	-52.3632	-1.4998	-52.4930	-1.1940
13	-52.2972	-1.6490	-52.4420	-1.3170
14	-52.2294	-1.7973	-52.3900	-1.4390
15	-52.1598	-1.9448	-52.3370	-1.5610
. . . .				
41	-50.6194	-4.4303	-50.5550	-4.5120
42	-50.5186	-4.5573	-50.4720	-4.6160
43	-50.4162	-4.6829	-50.3870	-4.7190
44	-50.3124	-4.8072	-50.3020	-4.8200
45	-50.2071	-4.9300	-50.2150	-4.9210
46	-50.1003	-5.0514	-50.1280	-5.0210
47	-49.9921	-5.1714	-50.0390	-5.1200
48	-49.8825	-5.2898	-49.9490	-5.2190
49	-49.7715	-5.4067	-49.8590	-5.3160
50	-49.6591	-5.5221	-49.7670	-5.4120
51	-49.5455	-5.6358	-49.6740	-5.5070
52	-49.4305	-5.7480	-49.5810	-5.6020
53	-49.3142	-5.8584	-49.4860	-5.6950
54	-49.1967	-5.9672	-49.3900	-5.7870
55	-49.0780	-6.0742	-49.2930	-5.8780
56	-48.9582	-6.1795	-49.1960	-5.9690
57	-48.8371	-6.2829	-49.0970	-6.0580
58	-48.7150	-6.3845	-48.9970	-6.1460
59	-48.5919	-6.4843	-48.8970	-6.2330
60	-48.4677	-6.5821	-48.7950	-6.3190
. . . .				
81	-45.6234	-8.1661	-46.4540	-7.8260
82	-45.4995	-8.2083	-46.3330	-7.8820
83	-45.3775	-8.2477	-46.2120	-7.9350
84	-45.2576	-8.2841	-46.0900	-7.9880
85	-45.1403	-8.3176	-45.9670	-8.0380
86	-45.0259	-8.3481	-45.8430	-8.0860
87	-44.9148	-8.3756	-45.7180	-8.1330
88	-44.8073	-8.4003	-45.5930	-8.1770
89	-44.7040	-8.4220	-45.4670	-8.2190
90	-44.6053	-8.4408	-45.3400	-8.2600
91	-44.5119	-8.4568	-45.2130	-8.2970
92	-44.4243	-8.4702	-45.0840	-8.3330
93	-44.3432	-8.4809	-44.9560	-8.3660
94	-44.2695	-8.4891	-44.8260	-8.3960
95	-44.2040	-8.4951	-44.6960	-8.4240
96	-44.1476	-8.4991	-44.5650	-8.4480
97	-44.1013	-8.5014	-44.4340	-8.4690
98	-44.0665	-8.5023	-44.3020	-8.4860
99	-44.0443	-8.5025	-44.1700	-8.4980
100	-44.0364	-8.5024	-44.0370	-8.5020

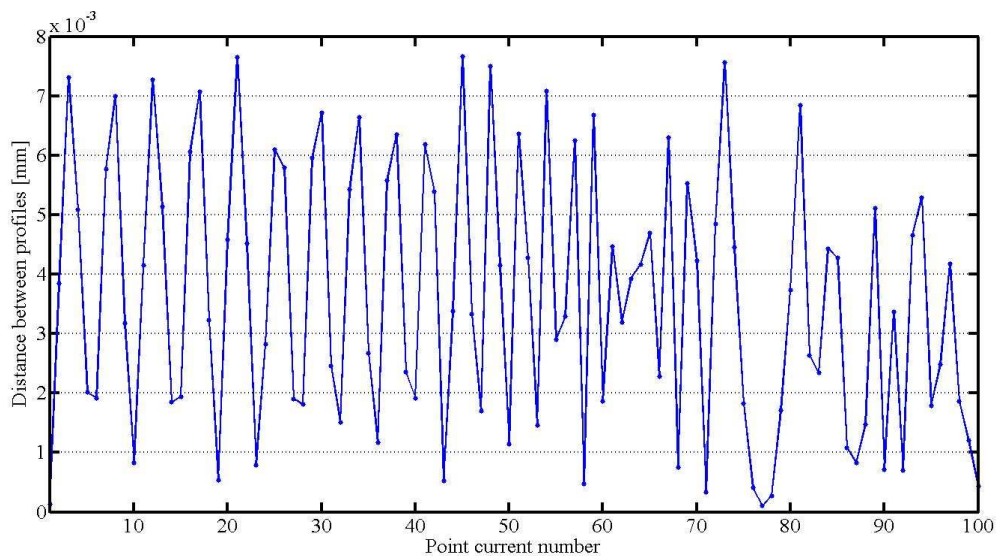**Fig. 6.** Rotating cutter profile.

Fig. 7. The distance between the profiles determined by using the graphical method and the analytical method.

At a first look, between the rotating cutter's profiles obtained by successively using the graphical method and the analytical method there are some differences. If the co-ordinates of their extreme points are nearly coincident, between the intermediary points co-ordinates seems to be differences up to several tenths of millimeter. In reality, we are not talking about the same points: the ones determined by the graphical method are equidistant along the tool profile, while the others were determined for equidistant values of θ parameter.

If we represent in the same reference system the profiles determined by both methods (Fig. 6) we may observe that, in fact, the differences are very small and they

cannot be discovered without using a considerable high zoom. The figure also shows the ball screw axial profile, beside the rotating cutter profile.

To make the things even clearer, we calculated the distances between each point determined by the graphical method and the nearest point from the analytical profile, by using a dedicated MATLAB application. If we intend to obtain realistic results, then the analytical profile must be divided into much more points. If we consider 100 points on the profile determined by using the graphical method and 1 000 points on the analytical profile, then the results

Table 2
Co-ordinates of the gearing line points

Point crt no	Graphical method	
	x [mm]	y [mm]
1	-52.9452	0.0000
2	-52.8099	-0.1029
3	-52.6670	-0.1951
4	-52.5176	-0.2761
5	-52.3625	-0.3456
6	-52.2027	-0.4035
7	-52.0391	-0.4497
8	-51.8727	-0.4844
9	-51.7043	-0.5077
10	-51.5347	-0.5199
. . . .		
46	-46.8550	2.9976
47	-46.7679	3.1436
48	-46.6824	3.2906
49	-46.5986	3.4386
50	-46.5163	3.5874
51	-46.4356	3.7371
52	-46.3564	3.8875
53	-46.2786	4.0387
54	-46.2023	4.1907
55	-46.1274	4.3433
. . . .		
91	-44.1069	10.1112
92	-44.0647	10.2759
93	-44.0231	10.4408
94	-43.9820	10.6058
95	-43.9415	10.7709
96	-43.9015	10.9362
97	-43.8620	11.1015
98	-43.8230	11.2671
99	-43.7845	11.4327
100	-43.7465	11.5984

are the ones depicted in Fig. 7. As it can be seen, the differences are smaller than a hundredth of millimeter, which fully shows the accuracy of the graphical method.

4.3. The gearing line

The study of the interference process between the reciprocal enveloped profiles is requiring to know the gearing line shape (given through co-ordinates of its points). The gearing line is defined as the locus of the contact points between two reciprocal enveloped profiles (in this case, the rotating cutter profile and the ball screw axial profile), expressed in the absolute reference system, *xy* (see Fig. 1).

The graphical method developed in CATIA in order to profile rotating cutters also enables to determine, similarly, the gearing line and to represent it. Hereby, the enwrapping circles with centers on the absolute system centrod must be drawn in order to find the locus of the contact points between tool and worked part. Their tangency points to the generated profile, expressed in the absolute system, result by applying the command *Translate*, with $R_{rs} \cdot \varphi$, to each substituting circle.

In Table 2 there are selectively presented the co-ordinates of the points from the gearing line in the case of the upper-approached numerical example, while in Fig. 8 we have the gearing line drawn in the same case, by joining the points whose co-ordinates were calculated.
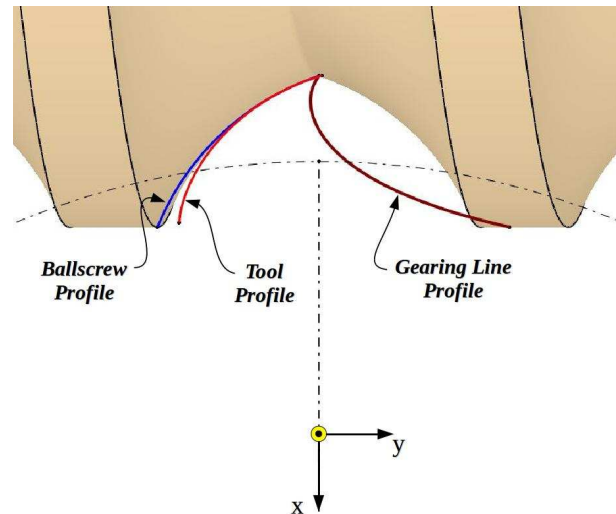

Fig. 8. The gearing line.

5. CONCLUSIONS

The graphical method here presented, developed in CATIA environment in order to profile the rotating cutter for generate by enwrapping the axial profile of a cylindrical helical surface with constant pitch is simple and it can be easily applied. We proved that the accuracy of this method in solving the approached problem is very high. The graphical method is also rapid and intuitive and it excludes the occurrence of application errors.

REFERENCES

[1] F.L. Litvin, A.G. Wang, R.F. Handschuh, *Computerized Generation and Simulation of Meshing and Contact of Spiral Bevel Gears with Improved Geometry*, Computational Methods for Applied Mechanical Engineering, 158, 1998, pp. 35–64.

[2] S.P. Radzevich, *Kinematics Geometry of Surface Machining*, CRC Press, London, 2008.

[3] N. Oancea, *Generarea suprafeţelor prin înfăşurare, Vol. I, II*, „Dunărea de Jos" Publishing House, Galaţi, 2004.

[4] N. Oancea, *Méthode numérique pour l'étude des surfaces enveloppées* (Numerical method for the study of enveloped surfaces), Mechanism and Machine Theory, vol. 31, no 7, 1996, pp. 957 – 972.

[5] V.G. Teodor, *Contribution to elaboration method for profiling tools which generate by enveloping*, Lambert Academic Publishing, 2010.

[6] S. Berbinschi, V. Teodor, N. Oancea, *A study on helical surface generated by the primary peripheral surfaces of ring tool*, available at http://www.springerlink.com /content/2560671327587214, accessed at 2014-05-07.

[7] S. Berbinschi , V. Teodor, N. Oancea, *3D graphical method for profiling tools that generate helical surfaces*, available at http://www.springerlink.com/content/g2118838 442xj351/ accessed at 2014-05-07.