# 3D KINEMATIC FIELDS STUDIES IN MILLING 

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#### Abstract

Comparing with other processes, milling and drilling have additional complexities arising from variation of the instantaneous geometric variables in machining and kinematic during operation [1,2]. In this study, the instantaneous variations of the kinematic and geometric cutting parameters are determined for a case of end milling with a milling tool with cutting inserts. The exact position of the insert in the space is determined according to the orientation's angles of the milling insert $\kappa_{r}, \gamma_{0}$ and $\lambda_{s}$ respectively the lead angle, rake angle and cutting edge inclination angle. For each inserts position, the kinematic torsor is determined for two representative points of the cutting edge. The first point is on the radius of the tool and the second is on the linear part of the cutting edge and depends on the depth of cut. The variation of the linear velocity due to the rotation of the tool and the feed rate, affects the cutting and the clearance angles and the instantaneous cutting velocity $V_{c, \text { orth }}$. The instantaneous feed is also determined by a numerical method starting from a geometric representation of the area scanned by the tool. The kinematic study is closed by a sensitivity study of instantaneous variation of cutting velocity in terms of depth of cut $a_{p}$ and feed per tooth $f_{z}$.


Key words: Milling, kinematic torsor, instantaneous cutting speed, instantaneous advance, sensitivity.

## 1. INTRODUCTION

Several researches have addressed the study of the position of the cutting edge during milling. Engine and al [1] defined mathematically the position of the cutting edge in a coordinate system linked to the center of cutting inserts. Two forms of cutting inserts were studied: rectangular and triangular convex. Saï and al [2] determine the position vector of the cutting edge for a monobloc tool in the case of a circular interpolation and a linear interpolation. There are not many analytical models about the kinematics study during milling process. Albert and al [3] determined the actions torsor at the tool tip using an experimental approach to highlight the cutting moments and the instantaneous variation of the kinematic parameters.

The present work consists to analytically analyze the three components of velocities determined in different points on the cutting edge taking into account the orientation of the cutting insert. A milling tool with inserts is studied and the cutting edge was divided in two parts: an area corresponding to the tool tip radius and a linear part where two extreme points have been chosen. The results help to analyse:
i) phenomena that may occur during cutting and
ii) tool-material interactions due to a large instantaneous variation of geometric and kinematic parameters.

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## Nomenclature:

$R_{0}$ : Coordinate systems linked to the tool
$R_{1,}, R_{2}, R_{3,} R_{4}$ : Coordinate systems linked to the insert.
$P_{s,} P_{f,} P_{r}:$ Planes linked to the insert.
$\kappa_{r}$ : Cutting edge angle between the ridge plane of the tool $P_{s}$ and the work plane $P_{f}$.
$\gamma_{0}$ : rake angle between the cutting face $\left(A_{\gamma}\right)$ and the reference plane $P_{r}$.
$\lambda_{s}$ : cutting edge inclination angle between the edge and the reference plane of the tool $P_{r}$.
$O_{0}, P$ : Tool center and tip of tool.
$\alpha_{0}, \varphi_{0}$ : Relief angle, Angle of variation.
$f_{z}, a_{p}$ : Feed per tooth, depth of cut.
$P_{1}, P_{2}$ : Points on the tool nose radius and on the linear part of the cutting edge.
$P_{2, \text { inf }}, P_{2, \text { sup }}$ : Lower and upper positions on the linear part of the cutting edge.
$x_{p \mathrm{i}}, y_{p i}, x_{p i+1}, y_{p i+1}$ : Coordinates of points $P_{i}$ and $P_{i+1}$. $\Delta_{\theta 1}, \mathcal{C}_{\theta 2}$ : Linear equation, equation for a curve.
$t_{2}, \theta, d_{\theta}$ : Time required for $P_{i+1}$ to be aligned with $P_{i}$, rotation angle, angle variation $\left(\theta_{2}-\theta_{1}\right)$.
$h_{m}, h_{m 1}$ : Instantaneous feed, instantaneous feed with rotation $\kappa_{r}$.
$\omega, V_{f}$ Rotation speed, feed velocity.
$V_{x,} V_{y,} V_{z}, V_{c, o r t h}$ : Three components of velocity, cutting velocity in orthogonal cutting configuration.
$h_{p 2}(\alpha)$ : Linear function.

## 2. TOOL GEOMETRY DESCRIPTION AND KINEMATIC PARAMETERS

A description of the geometry of the milling tool and its kinematic are presented in this paragraph. Several points on the insert are chosen to determine the influence


Fig. 1. Geometric configuration of the milling tool and Study points on the cutting edge.
of the cutting edge position on the interaction toolmaterial. During a milling operation, the geometric and kinematic cutting parameters of cutting are changing instantaneously during the tool rotation. These parameters will be presented along this paragraph.

### 2.1. Global tool geometry

Figure 1 shows the global tool geometry. The insert coordinate system $R_{1}\left(P, \overrightarrow{x_{1}}, \overrightarrow{y_{1}}, \overrightarrow{z_{1}}\right)$ rotates relative to the initial coordinate system $R_{0}\left(O_{0}, \overrightarrow{x_{0}}, \overrightarrow{y_{0}}, \overrightarrow{z_{0}}\right)$ linked to the tool center. The distance between $O_{0}$ and $P$ is equal to the tool radius. The rotation speed $\omega$ and the feed velocity $V_{f}$ generate the tool path. The insert is oriented in space with three angles $\kappa_{r}, \gamma_{0}$ and $\lambda_{s}$.

The initial coordinate system $R_{l}\left(P, \overrightarrow{x_{1}}, \overrightarrow{y_{1}}, \overrightarrow{z_{1}}\right)$ is linked to the insert in $P$ and two points $P_{1}$ and $P_{2}$ are defined respectively on the nose radius of the cutting edge and the linear part. Two extreme points on the linear part of the cutting edge are chosen to demonstrate the influence of the position on the velocities. The position of each point is determined by the function $h_{p i}(\alpha)$ which depends on the value of $\alpha \in[0,1]$.

$$
\begin{equation*}
h_{p 2}(\alpha)=\alpha \cdot\left(\frac{a_{p}}{\sin \left(\kappa_{r}\right) \cdot \cos \left(\lambda_{s}\right)}-r_{\varepsilon}\right) \tag{1}
\end{equation*}
$$

### 2.2. Kinematic parameters of modeling

Figure 2 shows, the variation of velocity components (linked to the insert local coordinate system $\left.\left(P, \overrightarrow{x_{1}}, \overrightarrow{y_{1}}, \overrightarrow{z_{1}}\right)\right)$ in the plane normal to the tool axis. The decomposition of the velocity depends on the feed rate and the rotation velocity of the tool.
$V_{c, o r t h}$ is the cutting speed used in the calculations of the elementary basic model in orthogonal cutting. The rotation and advance movements generate a new cutting plane. This latter plane is oriented relative to the initial coordinate system by $\varphi_{0}$, angle which depends on the decomposition of the cutting velocity.

In the milling process, the absolute value of $V_{c, \text { orth }}$ changes from a maximum value for $\theta=-180^{\circ}$, wherein the feed rate is added with the linear velocity $V_{x 1}$, to a minimum value for $\theta=0$ due to the subtraction of the same velocity.

For the different geometric parameters cited in Table 1 , the variation of $V_{x 1}$ in different points of cutting edge is represented in Fig. 3.


Fig. 2. Instantaneous variation of velocity upon rotation of the tool in the plane normal to the axis of the tool.


Fig. 3. Variation of $V_{x l}$ in terms of $\theta$.

## 3. INSTANTANEOUS VELOCITIES

The speed of each point on the edge of the tool depends on the insert orientation and the radius of tool. $R_{0}$ is an input modeling parameter representing the radial position of the reference tool point (point $P$ ). This distance remains constant and all orientations of the tool are relative to this point $P$.

These coordinates remain constant in any coordinate system related to the insert. Their position relative to the axis of the tool is given by the tool radius $R_{0}$ as presented in Fig. 1.

In this study, an angular position is considered after each rotation of $45^{\circ}$ of the tool. For each chosen position, the components of the instantaneous velocities and variations of cutting and clearance angles are determined.

### 3.1. Kinematic setting

The variation of the cutting edge angle $\kappa_{r}$ generates the rotation of the coordinate system $R_{1}$ related to the point $P$ of the insert in the reference tool plane (Fig. 5). In the coordinate system $R_{2}$, the distance of each point of the cutting edge relative to the tool axis depends for its radial position.

For the parameters given in table1, the variation of the difference of velocity $(d v)$ between the extreme points $P_{1}$ and $P_{2 \text {,sup }}$ in terms of depth of cut is presented in Fig. 4.

This variation increase with depth of cut, its value for $\kappa_{r}=45^{\circ}$ passes from $0.07 \mathrm{~m} . \mathrm{s}^{-1}$ to $0.31 \mathrm{~m} . \mathrm{s}^{-1}$. This difference becomes more important for the lower lead angles. The coordinate system $R_{3}$ is the same as $R_{2}$ and


Fig. 4. Variation of the difference of velocity $(d v)$ between the extreme points $P_{1}$ and $P_{2, \text { sup }}$ in terms of $a_{p}$ and $\kappa_{r}$


Fig. 5. Setting of the rotations $\kappa_{\mathrm{r}}, \gamma_{0}$ and $\lambda_{\mathrm{s}}$.
the insert rotates with respect to $z_{2}$ with $\gamma_{0}$ angle (cutting angle). This modelling was proposed to keep the same configuration of the orthogonal cutting shown in Fig. 2.

The cutting edge inclination angle $\lambda_{s}$ is due to the rotation of the system coordinate $R_{3}$ linked to the tool tip in the plane $P_{s}$.

### 3.2. Kinematic Torsor

The kinematic torsor is determined in different points of the cutting edge and for each position of the insert.

$$
\left[V_{P \in \text { Tool / workpiece }}\right]_{R_{1}}=\left\{\begin{array}{c}
\vec{\Omega}_{(\text {tool / workpiece })_{P}}  \tag{2}\\
\vec{V} \\
\left(O_{0} \in \text { tool / workpiece }\right)_{P}
\end{array}\right\}_{R_{1}}
$$

The kinematic torsor in point $P$ in the coordinate $R_{4}$ is obtained by rotations of this torsor from $R_{l}$ to $R_{4}$.

$$
\begin{gather*}
{\left[\mathrm{v}_{\text {Petool/workpiece }}\right]_{R_{4}}=} \\
\left\{\left[\begin{array}{c}
S\left(\lambda_{s}\right) \cdot C\left(\kappa_{r}\right) \cdot \omega \\
-C\left(\kappa_{r}\right) \cdot \omega \\
-C\left(\lambda_{s}\right) \cdot S\left(\kappa_{r}\right) \cdot \omega
\end{array}\right]\right.  \tag{3}\\
\left.\left[\begin{array}{c}
C\left(\lambda_{s}\right) \cdot\left(V_{f} \cdot C(\theta)-R_{0} \cdot \omega\right)+S\left(\lambda_{s}\right) \cdot C\left(\kappa_{r}\right) \cdot V_{f} \cdot S(\theta) \\
S\left(\kappa_{r}\right) \cdot V_{f} \cdot S(\theta) \\
S\left(\lambda_{s}\right) \cdot\left(V_{f} \cdot C(\theta)-R_{0} \cdot \omega\right)-C\left(\lambda_{s}\right) \cdot C\left(\kappa_{r}\right) \cdot V_{f} \cdot S(\theta)
\end{array}\right]\right\}_{R_{4}}
\end{gather*}
$$

with $C=\cos$ and $S=\sin$.
The velocity vector in $P_{1}$ is determined using the method of velocities transport:


Fig. 6. Variation of the angle $\varphi_{0}$ in terms of $a_{p}$.
$\vec{V}_{P_{1}, R_{4}}=\vec{V}_{P, R_{4}}+$
$\left[r_{\varepsilon} \cdot\left(\frac{2-\sqrt{2}}{2}\right) \cdot C\left(\gamma_{0}\right) \cdot C\left(\lambda_{s}\right) \cdot S\left(\kappa_{r}\right) \cdot \omega\right)-\left(r_{\varepsilon} \cdot\left(\frac{2-\sqrt{2}}{2}\right) \cdot C\left(\kappa_{r}\right) \cdot \omega\right)$
$-\left(r_{\varepsilon} \cdot\left(\frac{2-\sqrt{2}}{2}\right) \cdot S\left(\lambda_{s}\right) \cdot S\left(\kappa_{r}\right) \cdot \omega\right)-\left(r_{\varepsilon} \cdot\left(\frac{2-\sqrt{2}}{2}\right) \cdot S\left(\gamma_{0}\right) \cdot C\left(\lambda_{s}\right) \cdot S\left(\kappa_{r}\right) \cdot \omega\right)$
$\left(r_{\varepsilon} \cdot\left(\frac{2-\sqrt{2}}{2}\right) \cdot S\left(\gamma_{0}\right) \cdot C\left(\kappa_{r}\right) \cdot \omega\right)+\left(r_{\varepsilon} \cdot\left(\frac{2-\sqrt{2}}{2}\right) \cdot C\left(\gamma_{0}\right) \cdot S\left(\lambda_{s}\right) \cdot S\left(\kappa_{r}\right) \cdot \omega\right)$
Using the same method the velocity components of each point on the linear part of the cutting edge is determined:

$$
\vec{V}_{P_{2}, R_{4}}=\vec{V}_{P, R_{4}}+\left[\begin{array}{c}
-\left(r_{\varepsilon}+h_{p 2}(\alpha)\right) \cdot \cos \left(\kappa_{r}\right) \cdot \omega  \tag{5}\\
-\left(r_{\varepsilon}+h_{p 2}(\alpha)\right) \cdot \sin \left(\lambda_{s}\right) \cdot \sin \left(\kappa_{r}\right) \cdot \omega \\
0
\end{array}\right]_{R_{4}}
$$

### 3.3. Cutting angles analysis

In the orthogonal cutting configuration, the instantaneous variation of the velocities components generates the variation of the cutting and clearance angles by an angle $\varphi_{0}(\theta)$ during the milling operation. Upon rotation of the tool between $-180{ }^{\circ}$ to $0^{\circ}$, the cutting angle increases, in each position and the clearance angle decreases with the same value. The variation of $\varphi_{0}$ with depth of cut $\left(a_{p}\right)$ is represented in Fig. 6.

The $\varphi_{0}$ angle in $P_{1}$ does not depend on the depth of cut while its value is very sensitive to this parameter in point $P_{2}$. For $\theta=-270^{\circ}$, this angle goes from $0.28^{\circ}$ to $1.05^{\circ}$ in point $P_{2, \text { sup }}$. The variation of cutting angle between the two extremes points on the cutting edge can causes a strain between the different elements of the chip.

### 3.4. Kinematic results

For the different geometric parameters cited in Table 1, the variation of two components of velocity in the reference $R_{4}$ is determined.

The normal to the insert carried by $\overrightarrow{x_{4}}$ and the tangential component carried by $\overrightarrow{y_{4}}$, are presented as a function of the angle $\theta$ (Figs. 7 and 8 ).

Table 1
Geometric and kinematic parameters

| $\boldsymbol{\omega}$ <br> $\left(\right.$ rad. $\left.\mathbf{s}^{-1}\right)$ | $\kappa_{r}\left({ }^{\circ}\right)$ | $\boldsymbol{\gamma}\left({ }^{\circ}\right)$ | $\lambda_{s}\left({ }^{\circ}\right)$ | $a_{p}$ <br> $(\mathrm{~mm})$ | $f_{z}$ <br> $(\mathrm{~mm})$ | $R_{0}$ <br> $(\mathrm{~mm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | 45 | 6 | 6 | 2 | 0.2 | 25 |



Fig.7. Instantaneous velocity $V_{x 4}$ variation in the $R_{4}$ coordinate system.


Fig. 8. Instantaneous variation of the $V_{y 4}$ velocity in the $R_{4}$ coordinate system.

For $\theta \in\left[-180^{\circ}, 0\right]$ (tool in contact with work-piece), $V_{x 4}$ decreases during the combined movement of rotation and feed of the tool. The total difference between the maximum and minimum value of this velocity is equal to twice the feed velocity of the tool $V_{f}$. On the linear portion of the cutting edge, the absolute value of this velocity depends on the position of the point. For $\theta=0$ its value changes from $1.51 \mathrm{~m} . \mathrm{s}^{-1}$ in $P_{2, \text { inf }}$ to $1.61 \mathrm{~m} . \mathrm{s}^{-1}$ in $P_{2, \text { sup }}$.

The rotation of the insert with $\lambda_{s}$ angle generates the appearance of a new component of velocity carried by $\overrightarrow{y_{4}}$. Its variation for a full rotation of the tool is shown in Fig. 8.

The velocity $V_{y 4}$ is very sensible to the position of each point in zone 2 of the cutting edge. This value passes from $1 \mathrm{~mm} . \mathrm{s}^{-1}$ in $P_{2, \text { inf }}$ to $12 \mathrm{~mm} . \mathrm{s}^{-1}$ in point $P_{2, s u p}$.

The cutting edge inclination angle $\lambda_{s}$ generates an increase of the velocity carried by $\overrightarrow{z_{4}}$. Its average value rises, in $P_{2, \text { sup }}$, from $0.6 \mathrm{~mm} . \mathrm{s}^{-1}$ before rotation with $\lambda_{s}$ to a mean value $0.156 \mathrm{~m} . \mathrm{s}^{-1}$ after rotation (Fig. 9).


Fig.9. Instantaneous variation of the velocity $V_{y 4}$ in the $R_{4}$ coordinate system.


Fig. 10. Variation of $V_{c, \text { orrt }}$ with $\theta$ and $f_{z}$.


Fig. 11. Variation of $V_{c, \text { orrh }}$ with $\theta$ and $a_{\mathrm{p}}$.

### 3.5. Sensitivity of velocities variation with geometric parameters

For the parameters given in Table 1, the variation of cutting velocity $V_{c, \text { orth }}$ with $\theta$ angle is tested for different values of feed per tooth $f_{z}$ (Fig. 10).

Increasing the feed per tooth causes the increasing of feed velocity $V_{f}$. This latter variation of $V_{f}$ generates the decreasing of cutting velocity for $\theta=0$ and its increase for $\theta=-180^{\circ}$. The maximum variation of velocity for $f_{z}=$ 0.05 mm to $f_{z}=0.5 \mathrm{~mm}$ is equal to $5 \mathrm{~mm} . \mathrm{s}^{-1}$. This difference is low and the sensitivity of the variation of the cutting velocity to the feed per tooth is negligible.

The variation of cutting velocity $V_{c, \text { orth }}$ in terms of angle $\theta$ is tested for different values of depth of cut $a_{p}$ (Fig. 11).

The variation of cutting velocity $V_{c, \text { orth }}$ is very sensitive to depth of cut, its average value passe from $1.52 \mathrm{~m} . \mathrm{s}^{-1}$ for $a_{p}=0.5 \mathrm{~mm}$ to $1.79 \mathrm{~ms}^{-1}$ for $a_{p}=5 \mathrm{~mm}$.

## 4. CALCULATION OF INSTANTANEOUS FEED

Several studies are interested by the calculation of instantaneous feed, the first work of Martellotti [7, 8] considers the trochoïdal trajectory of tool. H.Z Li et al [9] have proposed a new approach of calculating for a linear trajectory of tool.

The displacement of each point is represented in Fig. 12. To determine the instantaneous feed of cutting, the variation angle $d_{\theta}$ between two successive rotations must be determine (in the case of tooth number higher than 1 this variation is determined between two successive tooth).


Fig. 12. Position of $P_{i}$ (rotation $i$ ) relative to $P_{i-1}$ (rotation $i-1$ ).
The position of point $P_{i}$ is determined from the equality between the linear equation $\Delta_{\theta 1}$ with the equation of $\mathcal{C}_{\theta 2}$. $\Delta_{\theta 1}$ corresponds to the first rotation and for an angle $\theta_{1}$. The second equation corresponds to the motion of the second rotation and for an angle $\theta_{2}$.

### 4.1. Method of calculating the instantaneous feed

The equation for the line $\Delta_{\theta 1}$ in the reference $\left(O_{i-1}, \vec{x}_{i-1}, \vec{y}_{i-1}, \vec{z}_{i-1}\right)$ related to the i-1 rotation is given by:

$$
\begin{equation*}
y_{p_{i-1, \theta_{1}}}=-x_{p_{i-1}} \cdot \tan \left(\theta_{1}\right) . \tag{6}
\end{equation*}
$$

The equation of the curve $\mathcal{C}_{\theta 2}$ in the reference $\left(O_{i-1}, \vec{x}_{i-1}, \vec{y}_{i-1}, \vec{z}_{i-1}\right)$ is given by:

$$
\begin{equation*}
y_{p_{i}, \theta_{2}}=x_{p_{i}} \cdot\left(\frac{V_{f} \cdot t_{2}}{R_{0} \cdot \cos \left(\theta_{2}\right)}-\tan \left(\theta_{2}\right)\right) . \tag{7}
\end{equation*}
$$

The equality between the two equations gives the relation between $\theta_{l}$ and $\theta_{2}$, as follows:

$$
\begin{equation*}
\operatorname{tg}\left(\theta_{2}\right)-\operatorname{tg}\left(\theta_{1}\right)=\frac{V_{f} \cdot t_{2}}{R_{0} \cdot \cos \left(\theta_{2}\right)} . \tag{8}
\end{equation*}
$$

Time $t_{2}$ corresponds to the time required for point $P_{i}$ to be aligned with $P_{i-1}$. This time is decomposed in three terms: time $t_{1}$, which represent the initial time corresponding to the position $P_{i-1}$ (in this configuration this term is equal to zero), $t_{\text {Itour }}$ which corresponds to one rotation of the tool and the time required for a rotation angle equal to $d_{\theta}$.

$$
\begin{equation*}
t_{2}=\frac{1}{N} \cdot\left(1+\frac{d_{\theta}}{2 \pi}\right) . \tag{9}
\end{equation*}
$$

Table 2
Instantaneous evolution of $\boldsymbol{\theta}_{\mathbf{2}}$ with $\boldsymbol{\theta}_{\mathbf{1}}$

| $\boldsymbol{\theta}_{\mathbf{1}}$ | $\boldsymbol{\theta}_{\mathbf{2}}$ with <br> $\boldsymbol{f}_{\boldsymbol{z}}=\mathbf{0 . 1}$ | $\boldsymbol{\theta}_{\mathbf{2}}$ with <br> $\boldsymbol{f}_{\boldsymbol{z}}=\mathbf{0 . 2}$ | $\boldsymbol{\theta}_{\mathbf{2}}$ with <br> $\boldsymbol{f}_{z}=\mathbf{0 . 3}$ |
| :---: | :---: | :---: | :---: |
| -180 | -179.78 | -179.55 | -179.32 |
| -202.5 | -202.29 | -202.06 | -201.86 |
| -225 | -224.84 | -224.67 | -224.52 |
| -247.5 | -247.41 | -247.33 | -247.24 |
| -270 | -269.99 | -269.99 | -269.99 |
| -292.5 | -292.59 | -292.68 | -292.76 |
| -315 | -315.17 | -315.34 | -315.48 |
| -337.5 | $-337,71$ | -337.91 | -338.14 |
| -360 | -360.23 | -360.46 | -360.69 |



Fig.13. Variation of $d_{\theta}$ with $\theta$.


Fig. 14. Change of instantaneous feed $h_{m}$ depending on $\theta$ for different values of $f_{z}$.

The equation (8) is solved numerically to determine for each rotation angle given $\theta_{1}$ the angle $\theta_{2}$. Table 2 present the instantaneous evolution of $\theta_{2}$ in terms of $\theta_{1}$ for three feed per tooth.

The variation of $d_{\theta}\left(d_{\theta}=\left\|\theta_{2}\right\|-\left\|\theta_{1}\right\|\right)$ in terms of $\theta$ is presented in Fig. 13 for different values of feed per tooth $f_{z}$. The value of $d_{\theta}$ is negative for $\theta$ between $-180^{\circ}$ and $-270^{\circ}$ and positive, for $\theta$ between $-270^{\circ}$ and $-360^{\circ}$.

The instantaneous feed $\left(h_{m}\right)$, for each insert angular position, is determined from the equations of motion (6 and 7). The general form of the instantaneous feed is:

$$
\begin{equation*}
h_{m}=\sqrt{\left(x_{p_{i}}-x_{p_{i-1}}\right)^{2}+\left(y_{p_{i}}-y_{p_{i-1}}\right)^{2}} . \tag{10}
\end{equation*}
$$

After simplification and with limited development near 0 of $\cos \left(d_{\theta}\right)$ and $\sin \left(d_{\theta}\right)$ the instantaneous feed becomes:

$$
\begin{equation*}
h_{m}=\sqrt{\left(R_{0} \cdot d_{\theta}\right)^{2}+\left(V_{f} \cdot t_{2}\left(d_{\theta}\right)\right)^{2}-2 \cdot R_{0} \cdot V_{f} \cdot t_{2} \cdot \cos \left(\theta_{1}\right) \cdot d_{\theta}} . \tag{11}
\end{equation*}
$$

### 4.2. Analysis

For a tool radius of 25 mm and an angular velocity of $60 \mathrm{rad} . \mathrm{s}^{-1}$, the variation of the instantaneous feed $h_{m}$ with $\theta$, for different feed per tooth, is shown in Fig. 14.

The instantaneous feed increases with the feed per tooth and its maximum value is equal to $f_{z}$. The instantaneous feed also depends on the insert orientation. The lead angle $\kappa_{r}$ causes the variation of its value after the rotation and it becomes:

$$
\begin{equation*}
h_{m 1}=\sin \left(\kappa_{r}\right) h_{m} \tag{12}
\end{equation*}
$$



Fig. 15. Instantaneous variation of the $V_{y 3}$ velocity in the $\mathcal{R}_{3}$ coordinate system.


Fig.16. Variation of the velocity $V_{y}$ with $\lambda_{s}$.

## 5. DISCUSSION

The instantaneous variations of cutting variables affect the kinematic behavior of chip and physics workpiece material behavior. 3D interactions may occur in each cutting zone during milling operation. Each insert orientation generates variation of one or more cutting parameters. Indeed the rotation $\kappa_{r}$ generates a small change in $V_{y}$ (Fig. 15) which does not affect the kinematics of cutting. Along edge of insert, the velocity component $V_{y}$ depends on the position of the considered point.

After rotation $\lambda_{s}$, the velocity $V_{y 4}$ increases starting by the tip to the point $P_{2, \text { sup }}$. This increasing generates a variation of the strain and the strain rate on the chip width and therefore the appearance of areas sheared more than others. If we consider the case of a rigid body, this speed difference causes the rotation of chip. The sense of velocity $V_{y}$ depends on the sign of angle $\lambda_{s}$. For a negative cutting edge inclination angle is positive (carried by $\overrightarrow{y_{4}}$ ) in $P_{2, \text { sup }}$. Its sense becomes negative (carried by $-\overrightarrow{y_{4}}$ ) for the positive cutting edge inclination angle (Fig. 16).

This non-uniform velocity distribution on the rake face can create, when chips are sliding, significant torques. The increasing of the absolute value of velocity carried by $\overrightarrow{z_{4}}$ generates a supplementary sliding of the chip carried by the cutting edge.

## 7. CONCLUSIONS

In this work a kinematic study during the milling process is presented. The first part of the article was devoted to the presentation of modeling data, the tool
geometry and the standard orientation angles of the insert.

The second part treated influence of the kinematics during cutting process on the instantaneous variation of the kinematic parameters (different components of velocity) and geometric parameters (instantaneous feed, cutting angles and clearance angle). From this last study, several interpretations are deduced:

- The rotation and advance movements of the tool generate a new cutting plane with cutting speed $V_{c, \text { orrh }}$. Its value changes from a maximum value for $\theta=-$ $180^{\circ}$ to a minimum value for $\theta=0$.
- The instantaneous variation of the velocities components generates the variation of the cutting and clearance angles during the milling operation.
- The rotation of the insert with $\lambda_{s}$ angle generates the appearance of a new component of velocity carried by $\overrightarrow{y_{4}}$. This velocity is very sensible to the position of each point in zone 2 of the cutting edge. The sense of velocity $V_{y}$ depends on the sign of angle $\lambda_{s}$.
- The cutting edge inclination angle $\lambda_{s}$ generates an increase of the velocity carried by $\overrightarrow{z_{4}}$.
To determine the results of a 3D model of milling (cutting forces, tangential forces and cutting moments) an elementary model of cutting must be used. Data from this model are instantaneous parameters determined along this study, this last step consist to enter this data in the elementary model taking into account the interaction 3 D when cutting.


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